Nonlinear Analysis of Cable Truss Structures under Static Loads

by

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Final Year Project Report submitted in partial fulfillment of the requirement of the Degree of

Bachelor of Science in Civil Engineering

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DECLARATION

I declare that the project report here submitted is original except for the source materials explicitly acknowledged and that this report as a whole, or any part of this report has not been previously and concurrently submitted for any other degree or award at the University of Macau or other institutions.

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Supervisor :  Prof. Guo-Kang Er
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Last but not least, I am deeply indebted to my family for all their constant support.
ABSTRACT

In this thesis, the linear and nonlinear cable truss equations of the biconvex and biconcave cable truss structures under four types of static loads, i.e. uniformly distributed vertical load applied over entire span, uniformly distributed vertical load applied on partial span, uniformly distributed vertical load applied over left half span and vertical point load, are derived using the compatibility equation suggested by Wang and Er (2013, 2014). Here, the effects of elastic deformations, temperature changes and elastic supports are taken into consideration. Computer programs are written, Newton–Raphson iteration method and Gaussian elimination method are applied to obtain the additional horizontal components of the top and bottom cable tension forces caused by the external force in the coupled nonlinear cable truss structures and linear cable truss structures, respectively. The additional vertical deflections caused by the external force also are analyzed according to the vertical equilibrium for the loaded cable trusses. The numerical results, i.e. the horizontal components of cable forces and vertical deflection of the symmetric and asymmetric cable trusses under various loads with different span-to-sag ratios, the horizontal components of cable forces and vertical deflection of the cable trusses with different loads obtained by linear and nonlinear analyses are presented, compared, and discussed. Through the investigation, it is shown that the linear cable truss theory has some limitations. When the deflection is large, which is normal for the flexible cable truss catenary structures, nonlinear cable truss theory is to be used in order to obtain relatively accurate results. Otherwise, the error can be too large to be acceptable with linear cable truss theory.
# TABLE OF CONTENTS

DECLARATION .................................................................................................................. I

APPROVAL FOR SUBMISSION......................................................................................... II

ACKNOWLEDGEMENTS ................................................................................................. III

ABSTRACT ..................................................................................................................... IV

TABLE OF CONTENTS .................................................................................................... V

LIST OF FIGURES ......................................................................................................... VIII

LIST OF SYMBOLS ....................................................................................................... XV

Chapter 1  Introduction ................................................................................................. 1

  1.1 Cable Truss and Its Analysis .................................................................................. 1

  1.2 Outline of This Thesis ......................................................................................... 2

Chapter 2  Literature Review ...................................................................................... 4

  2.1 Analyses of Cable Structures .............................................................................. 4

  2.2 Linear Analyses of Cable Truss Structures .......................................................... 5

  2.3 Nonlinear Analyses of Cable Truss Structures ..................................................... 7

Chapter 3  Linear Theory of Cable Trusses ................................................................. 10

  3.1 Initial Conditions for both Linear and Nonlinear Cable Truss Analysis ............... 10

  3.2 Vertical Deflections of the Linear Cable Trusses under Different Static Loads ...... 11

  3.3 Governing Equations for Linear Cable Trusses under Four Different Static Loading Cases .......................................................................................................................... 17
Chapter 4  Nonlinear Theory of Cable Trusses .................................................... 30

4.1 Vertical Deflections of Nonlinear Cable Trusses under Different Static Loads .......... 30

4.2 Governing Equations for the Nonlinear Cubic Cable Trusses under Four Different Static
Loading Cases ............................................................................................................ 33

4.3 Coefficients in the Governing Equations of Coupled Cubic Nonlinear Cable Trusses.. 46

4.4 Solution of Coupled Nonlinear Cubic Cable Truss Algebraic Equations ...................... 50

Chapter 5  Numerical Analysis of Linear and Nonlinear Cable Trusses............52

5.1 Different Span-to-Sag Ratios of Symmetric Cable Trusses................................. 52

5.1.1 Symmetric Cable Truss under Uniformly Distributed Vertical Load q=10kN/m ... 53

5.1.2 Symmetric Cable Truss under Uniformly Distributed Vertical Load q=10kN/m
from x=20m to x=40m ..................................................................................................... 57

5.1.3 Symmetric Cable Truss under Uniformly Distributed Vertical Load q=10kN/m over
Left Half Span .................................................................................................................. 60

5.1.4 Symmetric Cable Truss under Vertical Point Load P=200 kN at x=30m .......... 64

5.2 Different Span to Sag Ratios of Asymmetric Cable Trusses...................................... 69

5.2.1 Asymmetric Cable Truss under Uniformly Distributed Vertical Load q=10kN/m. 70

5.2.2 Asymmetric Cable Truss under Uniformly Distributed Vertical Load q=10kN/m
from x=20m to x=40m ..................................................................................................... 74

5.2.3 Asymmetric Cable Truss under Uniformly Distributed Vertical Load q=10 kN/m
over Left Half Span ......................................................................................................... 78

5.2.4 Asymmetric Cable Truss under Vertical Point Load P=200 kN at x=30m ........... 81
5.3 Different Loads Applied on Symmetric Cable Trusses ................................................. 86

5.3.1 Cable Truss under Various Uniformly Distributed Vertical Loads ...................... 86

5.3.2 Cable Truss under Various Uniformly Distributed Vertical Loads from x=20m to
x=40m ................................................................................................................................ 90

5.3.3 Cable Truss under Various Uniformly Distributed Vertical Loads over Left Half
Span ......................................................................................................................................... 93

5.3.4 Cable Truss under Various Vertical Point Loads at x=30m ................................... 97

Chapter 6 Conclusions and Recommendations ....................................................... 101

6.1 Conclusions .............................................................................................................. 101

6.2 Recommendations ................................................................................................. 104

References ..................................................................................................................... 105
LIST OF FIGURES

Figure 3.1 Geometries of Biconcave and Biconvex Cable Trusses .........................10
Figure 3.2 Four Different Typical Loading Cases ..................................................11
Figure 3.3 Sketch of the Vertical Equilibrium of Biconcave Cable Truss.................12
Figure 3.4 Displacements of Elements for Biconvex Cable Truss .........................17
Figure 5.1 Geometry of a Biconcave Symmetric Cable Truss under Uniformly Distributed Vertical Load .................................................................54
Figure 5.2 Horizontal Components of Cable Forces in Top Chord of Symmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios ..........54
Figure 5.3 Relative Errors of Ht of Linear Cable Truss vs. Span-to-sag Ratios ........54
Figure 5.4 Horizontal Components of Cable Forces in Bottom Chord of Symmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios .....55
Figure 5.5 Relative Errors of Hb of Linear Cable Truss vs. Span-to-sag Ratios .......55
Figure 5.6 Mid-span Vertical Deflections of Symmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios .....................................56
Figure 5.7 Relative Errors of Hb of Linear Cable Truss vs. Span-to-sag Ratios .......56
Figure 5.8 Geometry of a Biconcave Symmetric Cable Truss under Uniformly Distributed Vertical Load along Part of the Span.................................................57
Figure 5.9 Horizontal Components of Cable Forces in Top Chord of Symmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios ..........57
Figure 5.10 Relative Errors of Hb of Linear Cable Truss vs. Span-to-sag Ratios ......58
Figure 5.11 Horizontal Components of Cable Forces in Bottom Chord of Symmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios ...... 58
Figure 5.12 Relative Errors of Hb of Linear Cable Truss vs. Span-to-sag Ratios ...... 59
Figure 5.13 Mid-span Vertical Deflections of Symmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios .............................................. 59
Figure 5.14 Relative Errors of w of Linear Cable Truss vs. Span-to-sag Ratios ....... 60
Figure 5.15 Geometry of a Biconcave Symmetric Cable Truss under Uniformly Distributed Vertical Load over Left Half Span ............................................................. 60
Figure 5.16 Horizontal Components of Cable Forces in Top Chord of Symmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios ...... 61
Figure 5.17 Relative Errors of Ht of Linear Cable Truss vs. Span-to-sag Ratios ....... 61
Figure 5.18 Horizontal Components of Cable Forces in Bottom Chord of Symmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios ...... 62
Figure 5.19 Relative Errors of Hb of Linear Cable Truss vs. Span-to-sag Ratios ...... 62
Figure 5.20 Mid-span Vertical Deflections of Symmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios .............................................. 63
Figure 5.21 Relative Errors of w of Linear Cable Truss vs. Span-to-sag Ratios ....... 63
Figure 5.22 Geometry of a Biconcave Symmetric Cable Truss under Vertical Point Load .............................................................................................................................. 64
Figure 5.23 Horizontal Components of Cable Forces in Top Chord of Symmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios ...... 64
Figure 5.24 Relative Errors of Ht of Linear Cable Truss vs. Span-to-sag Ratios ....... 64

Figure 5.25 Horizontal Components of Cable Forces in Bottom Chord of Symmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios ...... 65

Figure 5.26 Relative Errors of Hb of Linear Cable Truss vs. Span-to-sag Ratios ...... 65

Figure 5.27 Mid-span Vertical Deflections of Symmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios .............................................. 66

Figure 5.28 Relative Errors of \( w \) of Linear Cable Truss vs. Span-to-sag Ratios ........ 66

Figure 5.29 Geometry of a Biconcave Asymmetric Cable Truss under Uniformly Distributed Vertical load .......................................................... 70

Figure 5.30 Horizontal Components of Cable Forces in Top Chord of Asymmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios ...... 70

Figure 5.31 Relative Errors of Ht of Linear Cable Truss vs. Span-to-sag Ratios ...... 71

Figure 5.32 Horizontal Components of Cable Forces in Bottom Chord of Asymmetric Cable Truss Obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios ..... 71

Figure 5.33 Relative Errors of Hb of Linear Cable Truss vs. Span-to-sag Ratios ...... 72

Figure 5.34 Mid-span Vertical Deflections of Asymmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios .............................................. 73

Figure 5.35 Relative Errors of \( w \) of Linear Cable Truss vs. Span-to-sag Ratios ........ 73

Figure 5.36 Geometry of a Biconcave Asymmetric Cable Truss under Uniformly Distributed Vertical Load along Part of the Span .............................................. 74
Figure 5.37 Horizontal Components of Cable Forces in Top Chord of Asymmetric Cable Truss Obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios ..... 74

Figure 5.38 Relative Errors for Ht of Linear Cable Truss vs. Span-to-sag Ratios...... 75

Figure 5.39 Horizontal Components of Cable Forces in Bottom Chord of Asymmetric Cable Truss Obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios ..... 75

Figure 5.40 Relative Errors for Hb of Linear Cable Truss vs. Span-to-sag Ratios...... 76

Figure 5.41 Mid-span Vertical Deflections of Asymmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios.............................................. 76

Figure 5.42 Relative Errors for w of Linear Cable Truss vs. Span-to-sag Ratios........ 77

Figure 5.43 Geometry of a Biconcave Asymmetric Cable Truss under Uniformly Distributed Vertical Load over Left Half Span ......................................................... 78

Figure 5.44 Horizontal Components of Cable Forces in Top Chord of Asymmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios ...... 78

Figure 5.45 Relative Errors for Ht of Linear Cable Truss vs. Span-to-sag Ratios...... 78

Figure 5.46 Horizontal Components of Cable Forces in Bottom Chord of Asymmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios ...... 79

Figure 5.47 Relative Errors for Hb of Linear Cable Truss vs. Span-to-sag Ratios...... 79

Figure 5.48 Mid-span Vertical Deflections of Asymmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios.............................................. 80

Figure 5.49 Relative Errors for w of Linear Cable Truss vs. Span-to-sag Ratios....... 80
Figure 5.50 Geometry of a Biconcave Asymmetric Cable Truss under Vertical Point Load

Figure 5.51 Horizontal Components of Cable Forces in Top Chord of Asymmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios

Figure 5.52 Relative Errors of Ht of Linear Cable Truss vs. Span-to-sag Ratios

Figure 5.53 Horizontal Components of Cable Forces in Bottom Chord of Asymmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios

Figure 5.54 Relative Errors for Hb of Linear Cable Truss vs. Span-to-sag Ratios

Figure 5.55 Mid-span Vertical Deflections of Asymmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios

Figure 5.56 Relative Errors of w of Linear Cable Truss vs. Span-to-sag Ratios

Figure 5.57 Geometry of a Biconcave Cable Truss under Various Uniformly Distributed Vertical Load

Figure 5.58 Horizontal Components of Cable Forces in Top Chord of Cable Truss obtained by Linear and Non-linear Analyses vs. Loads

Figure 5.59 Relative Errors for Ht of Linear Cable Truss vs. Loads

Figure 5.60 Horizontal Components of Cable Forces in Bottom Chord of Cable Truss obtained by Linear and Non-linear Analyses vs. Loads

Figure 5.61 Relative Errors for Hb of Linear Cable Truss vs. Loads

Figure 5.62 Mid-span Vertical Deflections of Cable Truss obtained by Linear and Non-linear Analyses vs. Loads
Figure 5.63 Relative Errors for $w$ of Linear Cable Truss vs. Loads ......................... 89

Figure 5.64 Geometry of a Biconcave Cable Truss under Uniformly Distributed
Vertical Load along part of the Span ........................................................................... 90

Figure 5.65 Horizontal Components of Cable Forces in Top Chord of Cable Truss
obtained by Linear and Non-linear Analyses vs. Loads.................................................. 90

Figure 5.66 Relative Errors for $H_t$ of Linear Cable Truss vs. Loads ......................... 91

Figure 5.67 Horizontal Components of Cable Forces in Bottom Chord of Cable Truss
obtained by Linear and Non-linear Analyses vs. Loads.................................................. 91

Figure 5.68 Relative Errors for $H_b$ of Linear Cable Truss vs. Loads ......................... 92

Figure 5.69 Mid-span Vertical Deflections of Cable Truss obtained by Linear and
Non-linear Analyses vs. Loads ...................................................................................... 92

Figure 5.70 Relative Errors for $w$ of Linear Cable Truss vs. Loads ......................... 93

Figure 5.71 Geometry of a Biconcave Cable Truss under Uniformly Distributed
Vertical Load over Left Half Span .................................................................................. 93

Figure 5.72 Horizontal Components of Cable Forces in Top Chord of Cable Truss
obtained by Linear and Non-linear Analyses vs. Loads.................................................. 94

Figure 5.73 Relative Errors for $H_t$ of Linear Cable Truss vs. Loads ......................... 94

Figure 5.74 Horizontal Components of Cable Forces in Bottom Chord of Cable Truss
obtained by Linear and Non-linear Analyses vs. Loads.................................................. 95

Figure 5.75 Relative Errors for $H_b$ of Linear Cable Truss vs. Loads ......................... 95
Figure 5.76 Mid-span Vertical Deflections of Cable Truss obtained by Linear and Non-linear Analyses vs. Loads

Figure 5.77 Relative Errors for $w$ of Linear Cable Truss vs. Loads

Figure 5.78 Geometry of a Biconcave Cable Truss under Vertical Point Load

Figure 5.79 Horizontal Components of Cable Forces in Top Chord of Cable Truss obtained by Linear and Non-linear Analyses vs. Loads

Figure 5.80 Relative Errors for $H_t$ of Linear Cable Truss vs. Loads

Figure 5.81 Horizontal Components of Cable Forces in Bottom Chord of Cable Truss obtained by Linear and Non-linear Analyses vs. Loads

Figure 5.82 Relative Errors for $H_b$ of Linear Cable Truss vs. Loads

Figure 5.83 Mid-span Vertical Deflections of Cable Truss obtained by Linear and Non-linear Analyses vs. Loads

Figure 5.84 Relative Errors for $w$ of Linear Cable Truss vs. Loads
**LIST OF SYMBOLS**

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>DEFINITION</th>
<th>UNIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_t$</td>
<td>Cross-section area of the top cable chord</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$A_b$</td>
<td>Cross-section area of the bottom cable chord</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$b_t$</td>
<td>Vertical distance from the top support to the x axis</td>
<td>m</td>
</tr>
<tr>
<td>$b_b$</td>
<td>Vertical distance from the bottom support to the x axis</td>
<td>m</td>
</tr>
<tr>
<td>$c$</td>
<td>Camber of the stabilizing bottom cable</td>
<td>m</td>
</tr>
<tr>
<td>$d_t$</td>
<td>Vertical distance from the highest point of the top chord to the x axis</td>
<td>m</td>
</tr>
<tr>
<td>$d_b$</td>
<td>Vertical distance from the lowest point of the bottom chord to the x axis</td>
<td>m</td>
</tr>
<tr>
<td>$E_t$</td>
<td>Modules of elasticity of the top chord</td>
<td>N/m$^2$</td>
</tr>
<tr>
<td>$E_b$</td>
<td>Modules of elasticity of the bottom chord</td>
<td>N/m$^2$</td>
</tr>
<tr>
<td>$H_t$</td>
<td>Total horizontal component of cable force in top chord due to the applied load</td>
<td>kN</td>
</tr>
<tr>
<td>$H_b$</td>
<td>Total horizontal component of cable force in bottom chord due to the applied load</td>
<td>kN</td>
</tr>
<tr>
<td>$H_{ot}$</td>
<td>Horizontal component of the pretension in the top chord</td>
<td>kN</td>
</tr>
<tr>
<td>$H_{ob}$</td>
<td>Horizontal component of the pretension in the bottom chord</td>
<td>kN</td>
</tr>
</tbody>
</table>
\( \Delta H_t \)  Additional horizontal component of cable force in top chord due to the applied load \( \text{kN} \)

\( \Delta H_b \)  Additional horizontal component of cable force in bottom chord due to the applied load \( \text{kN} \)

\( l \)  The span length of the cable truss \( \text{m} \)

\( L_{et} \)  Length of the unloaded top cable \( \text{m} \)

\( L_{eb} \)  Length of the unloaded bottom cable \( \text{m} \)

\( L_{Tt} \)  Length of the unloaded top cable due to temperature change \( \text{m} \)

\( L_{Tb} \)  Length of the unloaded bottom cable due to temperature change \( \text{m} \)

\( M \)  Bending moment \( \text{kN} \cdot \text{m} \)

\( P \)  Vertical point load \( \text{kN} \)

\( q \)  Vertical uniformly distributed load \( \text{kN/m} \)

\( Q \)  Shear force \( \text{kN} \)

\( s \)  Sag of the carrying top cable truss \( \text{m} \)

\( s_t \)  Initial length of the unloaded top cable truss element \( \text{m} \)

\( s_b \)  Initial length of the unloaded bottom cable truss element \( \text{m} \)

\( \bar{s}_t \)  New length of the top cable truss element due to displacement \( \text{m} \)
$\bar{s}_b$  New length of the bottom cable truss element due to m displacement

$u_t(l)$  Top longitudinal movement of the support at $x = l$ m

$u_t(0)$  Top longitudinal movement of the support at $x = 0$ m

$u_b(l)$  Bottom longitudinal movement of the support at $x = l$ m

$u_b(0)$  Bottom longitudinal movement of the support at $x = 0$ m

$w$  Additional vertical deflection m
Chapter 1 Introduction

1.1 Cable Truss and Its Analysis

Cable truss is a specific form of plane double-cable catenary system, anchored at both ends, composed of top and bottom continuous pre-stressed chords, and numerous light, rigid spacers placed in between.

Cable trusses can provide economical and considerable rigid solutions compared to single-cable system and have advantage in light self-weight, increased demand for large column-free spaces compared to other structures. So, in the recent decades, cable trusses have been widely used especially in the projects that require large coverage areas and roof support for large-span buildings.

One of the first influential cable trusses is Municipal Auditorium in Utica, N.Y which designed by the engineer Lev Zetlin and completed in 1959. Refer to Cuono (2012), it can be found that its 250 feet span roof is supported by cable trusses. For such a large span, the whole structure is much easier to be influenced by vibrations because of the slenderness of the high strength cables. And Zetlin overcome this problem by using the self-balancing roof system and cable trusses with top and bottom chords. In 1961, the Worker’s Gymnasium of Beijing was built using double-layer cable system with circular plan and the diameter of its roof is 90m. Further in 1962, a roof using a system of roof trusses known as the Jawerth biconcave cable truss system was placed over Johanneshov Ice Stadium in Stockholm. Watts Tower in Los Anglos used a transparent membrane to provide sun protection for occasional performance which suspended from radial cable trusses. And the roof of Stadium Oldenburg completed in 1995 consists of 14 anticlastic fabric panels as well suspended from cable trusses. These panels spans to the lowest point to ensure the required curvature for
pre-stressing. Besides, many other varieties throughout the whole world have been built incorporating the cable truss structure such as the Georgia Dome, Orlando International Airport, etc.

Another important use of cable truss system is in high walls especially the glass walls. As motioned by Lawson (2011), since the Radio France building in Paris used suspended glass supported by glass mullions in the 1960s, such glass wall started to be widely used. The architects and engineers are always finding lighter solutions to reduce the traditionally heavy members required to be used in high walls. And in 1986, the Museum of Science and Technology in Paris firstly executed a glass wall supported by cable truss which was a great success and the absence of horizontal supports makes the wall visually very light. As this cable truss curtain wall system has many advantages such as reflection, transparency, light wall surface, flexibility, strong adaptability for displacement of the structure, fast, convenient construction procedures with less material, no effect of weather conditions and energy saving. This type of cable truss supporting system becomes the most innovative design and has been popularly used in many famous structures like Louvre Pyramide Paris, University of Connecticut, New York Presbyterian Hospital, Harvard Medical School in Cambridge, and so on.

1.2 Outline of This Thesis

This thesis is composed of 6 chapters.

In chapter 2, a brief summary of the previous studies on the cable and cable truss structures is given with literature review.
In chapter 3, the governing equations of the linear cable trusses under four different static loads are derived, i.e. uniformly distributed vertical load applied over the entire span, uniformly distributed vertical load applied from $x=a$ to $x=b$ along the span, uniformly distributed vertical load applied over the left half of the span and vertical point load. The effects of temperature changes and elastic supports are also taken into considered during the analysis.

In chapter 4, the nonlinear governing equations of the cable trusses under four different static loads are derived, considering the geometrical nonlinearity and elastic deformation.

In chapter 5, computer programs are written to solve the linear and nonlinear algebraic equations for the cable truss. The numerical analysis is conducted for both symmetric and asymmetric cable trusses under four types of static loads. The horizontal components of cable forces and vertical deflections obtained by linear and nonlinear analyses versus various span-to-sag ratios and different loads are analyzed and presented in figures with discussions.

Some conclusions about the research and suggestions for further investigation are given in chapter 6.
Chapter 2 Literature Review

The application and analysis of cable structures started hundreds years ago. In recent decades, many studies have been carried out in order to analyze the linear and nonlinear behaviors of cable and cable truss structures.

2.1 Analyses of Cable Structures

Most of the earliest modern works emphasized to formulate the numerical methods for calculating the cable displacement. Michalos and Birnstiel (1962) presented one numerical method to figure out the load-induced displacements of a suspended cable. The effect of elastic deformation was considered during analysis. Janiszewski et al. (1962) outlined another method to determine the movements of a cable loaded in three dimensions. Through reviewing the computational procedure, an alternative convergence criterion for the iterative scheme was suggested in order to improve the calculation. Further, Siev (1964) presented the linearized equations for the displacement analysis of pre-stresses cable nets of any geometry subjected to any loadings with assumed elastic material properties. It showed that the calculation accuracy can be improved by the number of iterations. Besides, it also proposed that the analysis of displacements should be based on the final displacement positions as the large movement of the little rigid cable cannot be ignored in reason. Thornton and Birnstiel (1967) figured out another analytical procedure to determine the displacement of elastic three-dimensional suspension structures by considering the effects of temperature change and support movement during analysis which made it more close to real situation. O’Brien (1967) derived another numerical method which can be applied to any 3-D distributed loading acting over the full length or a part of the cable span. Krishna and Sparkes (1968) gave a ‘general nonlinear method’ which
could give a very good agreement with the model results to perform the deflection analyses.

Some studies, however, gave different solutions to some extent. Buchholdt (1969) provided a theoretically accurate method for calculating the forces and displacement of two dimensional pin-jointed skeletal pre-tensioned cable assemblies. Its structural mechanism is based on the minimization of the total potential energy and the method of steepest descant. Buchholdt (1970) given a theoretically accurate method used to calculate the forces and displacements of 2-D cable assemblies by minimizing the total potential energy and method of steepest descent was utilized. And it has the advantage compared with the previous theories that the method is always yield the correct solution. Because this method always converging towards the equilibrium position where the potential energy is minimum which. Saafan (1970) proposed an accurate finite cable deflection theory, and a more accurate representation of the material’s stress-strain relationship had been presented by using continuity and incremental load procedures during the analysis.

2.2 Linear Analyses of Cable Truss Structures

Regarding to cable-truss structures, many approximate linear static analyses have been carried out. The mathematical difficulties in linear analysis of cable truss structure have been explained and discussed in Greenberg (1970) and an accelerated convergence calculation technique with averaged stiffness coefficients was proposed. Eberhard et al. (1971) found that the procedure presented by Greenberg was efficient only for the simple examples which were all orthogonal nets and each consisting of two sets of four parallel cables spaced equal distance on center, however, it could not lead to a decrease in computation times for other cases. In addition, the elastic
analyses with nonlinear stress-strain relationship was correct only if all cables of the structure could be assumed to carry progressively increasing loads up to failure. Baron and Venkatesan (1971) considered the geometrical nonlinearity of the structures composed of elastic members capable of resisting axial forces. The proposed method could be effectively applied to analyze other nonlinear cable truss problems. The ASCE Subcommittee on Cable-Suspended Structures of the Task Committee on Special Structures firstly setup a particular section on the cable truss structure in 1971. Urelius and Fowler (1974) utilized an iterative routine by considering both geometric and conventional elastic stiffness to find the joint displacement and member forces of the cable trusses. The effects of individual design parameters on the behavior of cable trusses were examined by using a proposed computer program.

Rather than the simpler cable trusses that initially symmetric about both axes at mid-span, the analyses which covered both static and dynamic responses of the cable trusses that symmetric only about the vertical axis at mid-span were conducted by Irvine (1981). Neglecting all the second-order terms appeared in the differential equations, the linearized forms of the deflection and horizontal components of the cable forces were proposed. Krishna et al. (1982) considered the effects of various design parameters for different cable trusses with unyielding end supports under both symmetrical and asymmetrical loads. Though investigation, it concluded that the performance of the cable truss which connected at the center was the best compared with other cable trusses under the similar geometric and loading conditions. Raoof and Davies (2004) estimated the two-dimensional cable truss with different lay angles of the spiral strands theoretically by using more approximate no slip stiffness which could lead to practically significant reductions in vertical deflections of the cable truss under uniformly distributed load, and a computer program developed by Broughton
and Ndumbaros was used for the analysis.

2.3 Nonlinear Analyses of Cable Truss Structures

In the above studies, all second-order terms that appeared in the differential cable truss equations are neglected and little attention was paid to nonlinear analysis of cable trusses. In fact, however, significant nonlinearity would occur in the response of the cable truss with different initial geometries and material properties under various loads. Møllmann (1970) developed a nonlinear iteration method as well as an ALGOL program to obtain the joint displacement and member forces for the pre-stressed plane suspended cable structures. Irvine (1975) developed a method for the analysis of pre-stress, plan cable truss structure which was assumed to be elastic to determine the joint displacements and member forces corresponding to the equilibrium state, and the nonlinear effects as temperature effect, support movements were considered. Then, Panagiotopoulos (1976) derived two dual extremum principles: generalizations of the minimum of the potential and complementary energy with inequalities as subsidiary conditions. And it performed numerical analysis iteratively by using the decomposition techniques which applied in the algorithms of nonlinear optimization, in order to perform inelastic, stress-unilateral analysis of cable structures. In addition, a small strain elastic catenary element was proposed to solve both static and dynamic cable truss structure problems. During analysis, a finite element program was employed together with the available nonlinear codes based on a flexible iteration procedure presented by Jayaraman and Knudson (1981). Irvine (1981) carried out the work that focused and elaborated on the nonlinearity of cable trusses. Godbole et al. (1984) provided a general procedure which could be used to study the effect of flexible supports on the performance of suspended cable truss roofs by using finite element method. Kassimali and Parsi-Ferai doonian (1987) investigated the nonlinear
effects of large deformation, inelastic material properties and slackening on the cable truss behaviors and the ultimate strengths of pre-stressed cable truss. During the analysis, a method based on an Eulerian formulation was employed. Sultan et al. (2001) presented the general pre-stress stability conditions in accordance with the principle of virtual work for several tensegrity structures. And those conditions were expressed as a set of nonlinear equations which could be analytically solved. Sophianopoulos and Michaltsos (2001) proposed a simplified 2-mass, 3-DOF autonomous model utilizing the fully nonlinear dynamic stability analysis based on the energy method. And then, some mathematical difficulties caused by significant nonlinearity of the equations were overcome using modern commercial software.

Most of the recent nonlinear cable truss problems are analyzed by solving the derived nonlinear algebraic equations and numerical methods. Gasparini and Gautam (2002) studied the static behavior of three different cable systems by using geometrically nonlinear analyses. The nonlinear equilibrium equations were written in dimensionless forms and solved by Mathematica. Kanno et al. (2002) derived another special method for both friction and frictionless cable-joint analysis of nonlinear elastic cable trusses by utilizing a second-order cone programming analysis. And during the analysis, nonlinear stress behavior, large deformation and the material nonlinearity in tensile state were taken into consideration.

Lately, Kmet and Kokorudova (2006) modified the Irvine’s linearized form of the cable truss equations. By considering the effect of elastic deformation, temperature change as well as elastic supports of the biconvex and biconcave cable trusses under the vertical uniformly distributed load, the solutions for horizontal components of the cable forces and vertical deflections of the geometrically nonlinear cable trusses were
presented. The nonlinear cable truss solutions for more load types such as a vertical uniformly distributed load applied on part of the cable span were extended in Kmet and Kokorudova (2009). Through the investigation, it showed that the resulting horizontal components of the cable forces and vertical deflections obtained by the nonlinear analysis are of a great agreement with FEM results rather than those obtained by linear analysis. And this research has made good progress and great significance in the study of cable truss structure. Ma and Yao (2011) proposed a new type of annular tensile cable truss structures and studied the effect of pre-stress on static behavior of cable trusses. This study could be regarded as an effective reference for the further design and application of proposed new cable truss structure type. Recently, Wang and Er (2013, 2014) figured out the compatibility equation for the nonlinear analysis of the cable truss structure by considering the effect of the elastic deformation of the whole cable. This compatibility equation makes the governing equations of cables corrected and more reasonably formulated in theory. The corrected governing equations will be used for the studies of nonlinear cable truss behaviors in this thesis.
Chapter 3 Linear Theory of Cable Trusses

3.1 Initial Conditions for both Linear and Nonlinear Cable Truss Analysis

The following analysis of the cable trusses which are initially symmetric about the vertical axis at mid-span are based on the following assumptions:

1. The cables are completely flexible and they are working only in tension.

2. The profiles of both the top and bottom chords are assumed as parabolic curves.

3. Ignore all the small self-weight of the cables and the spacers.

4. The slopes of the chords remain relatively small during deformation.

5. Consider only the cable truss structure with spacers whose adjacent vertical elements are inextensible.

6. The small longitudinal movements of the chords freely occur.

The profiles of the geometries for both the biconcave and biconvex cable trusses are given in Figure 3.1.

![Figure 3.1 Geometries of Biconcave and Biconvex Cable Trusses](image-url)
As assumed before, the profiles of the bottom and top chords are both parabolic and they are expressed as follows.

\[
\begin{align*}
\text{Top chord: } z_t &= 4(d_t - b_t) \frac{x}{l} \left(1 - \frac{x}{l}\right) + b_t \\
\text{Bottom chord: } z_b &= 4(d_b - b_b) \frac{x}{l} \left(1 - \frac{x}{l}\right) + b_b
\end{align*}
\]

(3.1.1)

For biconcave cable truss, \( s = b_t - d_t \) and \( c = b_b - d_b \). While for the biconvex truss, \( s = d_b - b_b \) and \( c = d_t - b_t \).

### 3.2 Vertical Deflections of the Linear Cable Trusses under Different Static Loads

Four different load types as shown in Figure 3.2 are considered.

Figure 3.2 Four Different Typical Loading Cases
Figure 3.3 Sketch of the Vertical Equilibrium of Biconcave Cable Truss

With the vertical equilibrium of the cable truss as shown in Figure 3.3 referred to Irvine (1981), shear force on a cross section can be obtained as following.

\[ Q = (H_{ob} + \Delta H_b) \frac{d}{dx}(z_b + w) - (H_{ot} - \Delta H_t) \frac{d}{dx}(z_t - w) \]  

(3.2.1)

The initial internal equilibrium should be satisfied for the pre-stressed cable truss.

\[ H_{ob} \frac{dz_b}{dx} = H_{ot} \frac{dz_t}{dx} \]  

(3.2.2)

With Eq.(3.2.2), Eq.(3.2.1) can be written as

\[ Q = (H_{ob} + \Delta H_b) \frac{dw}{dx} + (H_{ot} - \Delta H_t) \frac{dw}{dx} + \Delta H_b \frac{dz_b}{dx} + \Delta H_t \frac{dz_t}{dx} \]  

(3.2.3)

In linear cable truss theory, \( \Delta H_b \frac{dw}{dx} \) and \( \Delta H_t \frac{dw}{dx} \) are neglected as they can lead to the third order terms in \( w \). Then Eq.(3.2.3) becomes

\[ Q = (H_{ob} + H_{ot}) \frac{dw}{dx} + \Delta H_b \frac{dz_b}{dx} + \Delta H_t \frac{dz_t}{dx} \]  

(3.2.4)
Integrating both sides of Eq.(3.2.4) gives the vertical deflection as

\[
w = \frac{1}{H_{ob} + H_{ot}} \left[ M - 4\Delta H_b (d_b - b_b) \frac{x}{l} \left(1 - \frac{x}{l}\right) - 4\Delta H_t (d_t - b_t) \frac{x}{l} \left(1 - \frac{x}{l}\right) \right] \tag{3.2.5}
\]

**Case 1: Cable Truss under Uniformly Distributed Vertical Load**

For the cable truss under a uniformly distributed vertical load \(q\) applied over the entire span, see Figure 3.2(a), the bending moment on a cross section is

\[
M = \frac{ql}{2} x \left(1 - \frac{x}{l}\right) \tag{3.2.6}
\]

Substitution Eq.(3.2.6) into Eq.(3.2.5) gives the vertical deflection as

\[
w = \frac{1}{H_{ob} + H_{ot}} \left[ \frac{ql}{2} \left( x - \frac{x^2}{l} \right) - 4\Delta H_b (d_b - b_b) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) - 4\Delta H_t (d_t - b_t) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) \right] \tag{3.2.7}
\]

**Case 2: Cable Truss under Uniformly Distributed Vertical Load from \(x=a\) to \(x=b\)**

For the cable truss under a uniformly distributed vertical load \(q\) from \(x = a\) to \(x = b\) along the span, see Figure 3.2(b), the bending moment on a cross section is

\[
M = \frac{q}{2l} (b - a)^2 + \frac{q}{l} (b - a)(l - b) \tag{3.2.8}
\]

for \(x \in (0, a),\)
\[ M = \frac{qx}{2l}(b-a)^2 + \frac{qx}{l}(b-a)(l-b) - \frac{q}{2}x^2 + qax - \frac{qa^2}{2} \quad (3.2.9) \]

for \( x \in (a, b) \), and

\[ M = \frac{qx}{2l}(b-a)^2 + \frac{qx}{l}(b-a)(l-b) + \frac{qb^2}{2} - qx(b-a) - \frac{qa^2}{2} \quad (3.2.10) \]

for \( x \in (b, l) \).

Substitution Eqs. (3.2.8), (3.2.9), and (3.2.10) into Eq. (3.2.5) gives the vertical deflection as

\[
W = \frac{1}{H_{ob} + H_{of}} \left[ \frac{ qx }{ 2l } (b-a)^2 + \frac{ q }{ l } (b-a)(l-b) - \frac{ q }{ 2 } x^2 + qax - \frac{ qa^2 }{ 2 } + \frac{ qx }{ l } (b-a)(l-b) - 4 \Delta H_b (d_b-b) \right]
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]

\[
\left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right) - 4 \Delta H_b (d_b-b) \left( \frac{ x }{ l } - \frac{ x^2 }{ l^2 } \right)
\]
Case 3: Cable Truss under Uniformly Distributed Vertical Load over Left Half Span

For cable truss under a uniformly distributed vertical load $q$ applied over the left half of the span from $x = 0$ to $x = l/2$, see Figure 3.2(c), the bending moment is

$$M = ql \left( \frac{3x}{8} - \frac{x^2}{2l} \right)$$

(3.2.14)

for $x \in (0,l/2)$, and

$$M = \frac{ql}{8} (l-x)$$

(3.2.15)

for $x \in (l/2,l)$.

Substituting Eqs.(3.2.14) and (3.2.15) into Eq.(3.2.5), the vertical deflections can be got as

$$w = \frac{1}{H_{ob} + H_{ot}} \left[ ql \left( \frac{3x}{8} - \frac{x^2}{2l} \right) - 4\Delta H_b \left( d_b - b_b \right) \frac{x}{l} \left( 1 - \frac{x}{l} \right) - 4\Delta H_t \left( d_t - b_t \right) \frac{x}{l} \left( 1 - \frac{x}{l} \right) \right]$$

(3.2.16)

for $x \in (0,l/2)$, and

$$w = \frac{1}{H_{ob} + H_{ot}} \left[ ql (l-x) - 4\Delta H_b \left( d_b - b_b \right) \frac{x}{l} \left( 1 - \frac{x}{l} \right) - 4\Delta H_t \left( d_t - b_t \right) \frac{x}{l} \left( 1 - \frac{x}{l} \right) \right]$$

(3.2.17)

for $x \in (l/2,l)$. 
Case 4: Cable Truss under Vertical Point Load at $x=e$

For cable truss under a vertical point load $P$ applied at $x = e$ along the span, see Figure 3.2(d), the bending moment is

$$M = Px \left(1 - \frac{e}{l}\right)$$  \hspace{1cm} (3.2.18)

for $x \in (0, e)$, and

$$M = Pe \left(1 - \frac{x}{l}\right)$$  \hspace{1cm} (3.2.19)

for $x \in (e, l)$.

Substitution Eqs. (3.2.18) and (3.2.19) into Eq. (3.2.5) gives the vertical deflection as

$$w = \frac{1}{H_{ob} + H_{ot}} \left[ Px \left(1 - \frac{e}{l}\right) - 4\Delta H_b (d_b - b_b) \frac{x}{l} \left(1 - \frac{x}{l}\right) - 4\Delta H_t (d_t - b_t) \frac{x}{l} \left(1 - \frac{x}{l}\right) \right]$$  \hspace{1cm} (3.2.20)

for $x \in (0, e)$, and

$$w = \frac{1}{H_{ob} + H_{ot}} \left[ Pe \left(1 - \frac{x}{l}\right) - 4\Delta H_b (d_b - b_b) \frac{x}{l} \left(1 - \frac{x}{l}\right) - 4\Delta H_t (d_t - b_t) \frac{x}{l} \left(1 - \frac{x}{l}\right) \right]$$  \hspace{1cm} (3.2.21)

for $x \in (e, l)$. 

3.3 Governing Equations for Linear Cable Trusses under Four Different Static Loading Cases

The displacements for a biconvex cable truss elements presented by Kmet and Kokorudova (2009) are shown in Figure 3.4. Before the loading is applied, \( ds_t = dx^2 + dz_t^2 \), \( ds_b = dx^2 + dz_b^2 \). After deformation, \( d\bar{s}_t = (dx - du_t)^2 + (dz_t - dw)^2 \), \( d\bar{s}_b = (dx + du_b)^2 + (dz_b + dw)^2 \).

![Figure 3.4 Displacements of Elements for Biconvex Cable Truss](image)

The difference of the initial length and deformed one of the bottom cable truss element is formulated as following using the compatibility equation suggested by Wang and Er (2013, 2014).

\[
\bar{s}_b - s_b = \int_{0}^{l} (d\bar{s}_b - ds_b) = \int_{0}^{l} \left[ \sqrt{\left(1 + \left(\frac{du_b}{dx}\right)^2\right)^2 + \left(\frac{dz_b}{dx} + \frac{dw}{dx}\right)^2} - \sqrt{1 + \left(\frac{dz_b}{dx}\right)^2} \right] dx
\]  

(3.3.1)
Using Taylor series, the term of \( d\bar{s}_b - ds_b \) can be expended to include second order terms in \( w \),

\[
 d\bar{s}_b - ds_b = \frac{d\bar{s}_b - ds_b}{ds_b}
\]

\[
 = \left( 1 + \frac{du_b}{dx} \right)^2 + \left( \frac{dz_b}{dx} + \frac{dw}{dx} \right)^2 - 1 \quad ds_b
\]

\[
 = \left( 1 + \frac{du_b}{dx} \right)^2 + 2 \frac{du_b}{dx} + 2 \frac{dz_b}{dx} \frac{dw}{dx} + \left( \frac{dw}{dx} \right)^2 - 1 \quad ds_b
\]

\[
 = \frac{1}{2} \left( \frac{du_b}{dx} \right)^2 + \frac{du_b}{dx} \frac{dz_b}{dx} \frac{dw}{dx} + \frac{1}{2} \left( \frac{dw}{dx} \right)^2
\]

\[
(3.3.2)
\]

Substitution Eq.(3.3.2) into eq.(3.3.1) gives

\[
 \bar{s}_b - s_b = \int_0^l \left[ \frac{du_b}{dx} + \frac{dw}{dx} \frac{dz_b}{dx} + \frac{1}{2} \left( \frac{du_b}{dx} \right)^2 + \frac{1}{2} \left( \frac{dw}{dx} \right)^2 \right] dx
\]

\[
(3.3.3)
\]

where the term \( \frac{1}{2} \left( \frac{du_b}{dx} \right)^2 \) can be ignored because it is relatively small compared to other terms.

In linear cable truss theory, the term of \( \frac{1}{2} \left( \frac{dw}{dx} \right)^2 \) is not considered.
Using Hooke’s law, the difference of the initial length and deformed one of the bottom cable truss element can be expressed as

\[ \bar{s}_b - s_b = \int \frac{\Delta H_b}{E_b A_b} \frac{ds_b}{dx} ds_b \]  
(3.3.4)

Substitution Eq.(3.3.4) into Eq.(3.3.3) gives

\[ \int \frac{\Delta H_b}{E_b A_b} \left( \frac{ds_b}{dx} \right)^2 dx = \int \left[ \frac{du_b}{dx} + \frac{dw}{dx} \frac{ds_b}{dx} \right] dx \]  
(3.3.5)

Similarly for the top chord of the cable truss,

\[ \int \frac{\Delta H_t}{E_t A_t} \left( \frac{ds_t}{dx} \right)^2 dx = \int \left[ \frac{du_t}{dx} + \frac{dw}{dx} \frac{ds_t}{dx} \right] dx \]  
(3.3.6)

Finally, the following governing equations are obtained.

\[ \frac{\Delta H_t}{E_t A_t} L_t = u_t (l) - u_t (0) + \int_0^l \left( \frac{dw}{dx} \frac{ds_t}{dx} \right) dx \]  
(3.3.7)

\[ \frac{\Delta H_b}{E_b A_b} L_b = u_b (l) - u_b (0) + \int_0^l \left( \frac{dw}{dx} \frac{ds_b}{dx} \right) dx \]

The coefficients \( L_{eb}, L_{et} \) are specified by

\[ L_o = \int_0^l \left( \frac{ds_t}{dx} \right)^2 dx \cong l \left[ 1 + 16 \left( \frac{dt - bt}{l} \right)^2 \right] \]  
(3.3.8)

\[ L_{eb} = \int_0^l \left( \frac{ds_b}{dx} \right)^2 dx \cong l \left[ 1 + 16 \left( \frac{db - bb}{l} \right)^2 \right] \]

where

\[ ds_t^2 = dx^2 + dz_t^2, ds_b^2 = dx^2 + dz_b^2, \]

\[ \frac{ds_t}{dx}, \frac{ds_b}{dx} = \sqrt{1 + \left( \frac{dz_t}{dx} \right)^2}, \frac{ds_b}{dx} = \sqrt{1 + \left( \frac{dz_b}{dx} \right)^2}. \]

Only the first two terms of the binomial series are considered.
In addition, consider the effect of a uniform temperature change. The difference of the initial length and deformed one of the bottom element should be expressed as

$$s_b - s_h = \int \left( \frac{\Delta H_b}{E_s A_b} + \alpha \Delta T_b \right) \frac{ds_b}{dx} \, dx$$  \hspace{1cm} (3.3.9)

Substituting Eq.(3.3.9) into Eq.(3.3.3) gives

$$\frac{\Delta H_t}{E_s A_t} L_{et} + \alpha \Delta T_t L_{et} = u_t(l) - u_t(0) + \int_0^l \left( \frac{dw}{dx} \frac{dz}{dx} \right) \, dx$$  \hspace{1cm} (3.3.10)

$$\frac{\Delta H_t}{E_s A_b} L_{eb} + \alpha \Delta T_b L_{eb} = u_b(l) - u_b(0) + \int_0^l \left( \frac{dw}{dx} \frac{dz}{dx} \right) \, dx$$

As before, the coefficients $L_{eb}, L_{et}$ are specified by

$$L_{et} = \int_0^l \left( \frac{ds_t}{dx} \right)^2 dx \geq l \times \left[ 1 + \frac{16}{3} \left( \frac{dt}{l} \right)^2 \right]$$  \hspace{1cm} (3.3.11)

$$L_{eb} = \int_0^l \left( \frac{ds_b}{dx} \right)^2 dx \geq l \times \left[ 1 + \frac{16}{3} \left( \frac{db}{l} \right)^2 \right]$$

where $ds_t^2 = dx^2 + dz_t^2, ds_b^2 = dx^2 + dz_b^2, ds_t = \sqrt{1 + \left( \frac{dz_t}{dx} \right)^2}, ds_b = \sqrt{1 + \left( \frac{dz_b}{dx} \right)^2}.$

After making appropriate substitution and performing the necessary integration, the identical coupled system of cable truss Eqs.(3.3.10) can be used for obtaining $\Delta H_b$ and $\Delta H_t$ by solving the simultaneous algebraic equations as following.

$$c_{h1} \Delta H_b + c_{h2} \Delta H_t + c_{h3} = 0$$

$$c_{i1} \Delta H_b + c_{i2} \Delta H_t + c_{i3} = 0$$  \hspace{1cm} (3.3.12)
Case 1: Cable Truss under Uniformly Distributed Vertical Load

For cable truss under a uniformly distributed vertical load applied over the entire span, as \( \frac{dw}{dx} \) is continuous along the span, taking integration by parts for Eqs.(3.3.10) gives

\[
\frac{\Delta H_{L_{w_c}}}{E_i A_i} + \alpha \Delta T_i L_{T_0} = -\int_0^l \left( \frac{d^2 z_t}{dx^2} w \right) dx + \left[ \frac{dz_t}{dx} w \right]_0^l + B_t
\]

(3.3.13)

\[
\frac{\Delta H_{L_{w_b}}}{E_b A_b} + \alpha \Delta T_b L_{T_0} = -\int_0^l \left( \frac{d^2 z_b}{dx^2} w \right) dx + \left[ \frac{dz_b}{dx} w \right]_0^l + B_b
\]

where \( B_b = u_b(l) - u_b(0) \) and \( B_t = u_t(l) - u_t(0) \); for unmovable supports \( B_b = 0 \) and \( B_t = 0 \). The terms \( \left[ \frac{dz_t}{dx} w \right]_0^l \) and \( \left[ \frac{dz_b}{dx} w \right]_0^l \) equal zero.

In Eqs.(3.3.13), the terms \( \frac{d^2 z_t}{dx^2} \) and \( \frac{d^2 z_b}{dx^2} \) can be obtained by taking the second derivative of \( z_t \) and \( z_b \) expressed by Eqs.(3.1.1) with respect to \( x \).

\[
\frac{d^2 z_t}{dx^2} = 4(d_t - b_t) \left( -\frac{2}{l^2} \right)
\]

(3.3.14)

\[
\frac{d^2 z_b}{dx^2} = 4(d_b - b_b) \left( -\frac{2}{l^2} \right)
\]

Then, substituting Eqs.(3.3.14) and (3.2.7) into Eqs.(3.3.13) gives

\[
\frac{\Delta H_{L_{w_c}}}{E_i A_i} + \alpha \Delta T_i L_{T_0} = -C(d_t - b_t) + B_t
\]

(3.3.15)

\[
\frac{\Delta H_{L_{w_b}}}{E_b A_b} + \alpha \Delta T_b L_{T_0} = -C(d_b - b_b) + B_b
\]

where

\[
C = \left[ -4 \left( -\frac{2}{l^2} \right) \frac{1}{\Delta H_b + \Delta H_t} \int_0^l \frac{q l}{2} \left( x - \frac{x^2}{l} \right) - 4 \Delta H_b (d_b - b_b) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) - 4 \Delta H_t (d_t - b_t) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) \right] dx
\]
Through multiplying \((H_{ob} + \Delta H_b)\) on both sides of the above equations, performing necessary integration and rewriting them in a more visible way, two linear coupled cable truss equations can be obtained for this case and they are similar to Eqs.(3.3.12).

**Case 2: Cable Truss under Uniformly Distributed Vertical Load from x=a to x=b**

For cable truss under a uniformly distributed vertical load \(q\) applied from \(x = a\) to \(b\) along the span, taking integration by parts for Eqs.(3.3.10) gives

\[
\frac{\Delta H_{ob}}{E_{ob}A_{ob}} + \alpha \Delta T_{ob} = \int_{a}^{b} \left\{ \frac{d^2 z}{dx^2} w_1 \right\} dx - \int_{a}^{b} \left\{ \frac{d^2 z}{dx^2} w_2 \right\} dx - \int_{a}^{b} \left\{ \frac{d^2 z}{dx^2} w_3 \right\} dx + \left[ \frac{dz}{dx} w_1 \right]_a^b + \left[ \frac{dz}{dx} w_2 \right]_a^b + \left[ \frac{dz}{dx} w_3 \right]_a^b + B_i
\]

\[
\frac{\Delta H_{ob}}{E_{ob}A_{ob}} + \alpha \Delta T_{ob} = \int_{a}^{b} \left\{ \frac{d^2 z}{dx^2} w_1 \right\} dx - \int_{a}^{b} \left\{ \frac{d^2 z}{dx^2} w_2 \right\} dx - \int_{a}^{b} \left\{ \frac{d^2 z}{dx^2} w_3 \right\} dx + \left[ \frac{dz}{dx} w_1 \right]_a^b + \left[ \frac{dz}{dx} w_2 \right]_a^b + \left[ \frac{dz}{dx} w_3 \right]_a^b + B_b
\]

where \(w_1 \in (0, a), w_2 \in (a, b)\) and \(w_3 \in (b, l)\). The terms \(\left[ \frac{dz}{dx} w_1 \right]_0^a + \left[ \frac{dz}{dx} w_2 \right]_a^b + \left[ \frac{dz}{dx} w_3 \right]_a^b\) and \(\left[ \frac{dz}{dx} w_2 \right]_0^a + \left[ \frac{dz}{dx} w_2 \right]_a^b + \left[ \frac{dz}{dx} w_3 \right]_a^b\) equal zero.

Then, substitution Eqs.(3.3.14), (3.2.11), (3.2.12) and (3.2.13) into Eqs.(3.3.16) gives

\[
\frac{\Delta H_{ob}}{E_{ob}A_{ob}} + \alpha \Delta T_{ob} = -C (d_i - b_i) + B_i
\]

\[
\frac{\Delta H_{ob}}{E_{ob}A_{ob}} + \alpha \Delta T_{ob} = -C (d_b - b_b) + B_b
\]

where
Through multiplying \((H_{ob} + \Delta H_b)\) on both sides of the above equations, performing necessary integration and rewriting them in a more visible way, two linear coupled cable truss equations can be obtained for this case and they are similar to Eqs.(3.3.12).

**Case 3: Cable Truss under Uniformly Distributed Vertical Load over Left Half Span**

For the cable truss under a uniformly distributed vertical load \(q\) applied over the left half of the span from \(x = 0\) to \(x = l/2\), taking integration by parts for Eqs.(3.3.10) gives

\[
\frac{\Delta H_b L_{lb}}{E_b A_b} + \alpha \Delta T_b L_{lb} = -\int_0^{l/2} \left( \frac{d^2 z_{b_w}}{dx^2} w_1 \right) dx - \int_{l/2}^l \left( \frac{d^2 z_{b_w}}{dx^2} w_2 \right) dx + \left[ \frac{dz_{b_w}}{dx} w_1 \right]_0^l + \left[ \frac{dz_{b_w}}{dx} w_2 \right]_{l/2}^{l/2} \frac{l}{2} + B_b
\]

\[
\frac{\Delta H_{b_T} L_{lb}}{E_{b_T} A_{b_T}} + \alpha \Delta T_{b_T} L_{lb} = -\int_0^{l/2} \left( \frac{d^2 z_{T_w}}{dx^2} w_1 \right) dx - \int_{l/2}^l \left( \frac{d^2 z_{T_w}}{dx^2} w_2 \right) dx + \left[ \frac{dz_{T_w}}{dx} w_1 \right]_0^l + \left[ \frac{dz_{T_w}}{dx} w_2 \right]_{l/2}^{l/2} \frac{l}{2} + B_{b_T}
\]

where \(w_1 \in (0, l/2)\) and \(w_2 \in (l/2, l)\). The terms \(\left[ \frac{dz_{w_1}}{dx} w_1 \right]_0^l + \left[ \frac{dz_{w_2}}{dx} w_2 \right]_{l/2}^{l/2} \frac{l}{2}\) and
\[
\left[ \frac{dz_b}{dx} w_1 \right]_0^\frac{l}{2} + \left[ \frac{dz_b}{dx} w_2 \right]_0^\frac{l}{2} = 0.
\]

Then, substituting Eqs. (3.3.14), Eq. (3.2.16) and (3.2.17) into Eqs. (3.3.18) gives

\[
\frac{\Delta H_I L_{eb}}{E_b A_b} + \alpha \Delta T_e L_{et} = -C\left( d_i - b_i \right) + B_i \tag{3.3.19}
\]

\[
\frac{\Delta H_I L_{eb}}{E_b A_b} + \alpha \Delta T_b L_{et} = -C\left( d_b - b_b \right) + B_b
\]

where

\[
C = \left[ -4\left( -\frac{2}{l^2} \right) \frac{1}{\alpha H_b + \Delta H_t} \right] \left[ \int_0^{\frac{l}{2}} \left( 3x - \frac{x^2}{8} - \frac{x}{l} \right) \left( 1 - \frac{x}{l} \right) - 4\Delta H_b \left( d_b - b_b \right) \right] d\alpha
\]

Through multiplying \((H_{ob} + \Delta H_b)\) on both sides of the above equations, performing necessary integration and rewriting them in a more visible way, two linear coupled cable truss equations can be obtained for this case and they are similar to Eqs. (3.3.12).

**Case 4: Cable Truss under Vertical Point Load at \(x = e\)**

For cable truss under a vertical point load \(P\) applied at \(x = e\), taking integration by parts for Eqs. (3.3.10) gives

\[
\frac{\Delta H_I L_{eb}}{E_b A_b} + \alpha \Delta T_e L_{et} = -\int_0^e \left[ \frac{d^2 z_e}{dx^2} \right] w_1 \ dx - \int_0^e \left[ \frac{d^2 z_e}{dx^2} \right] W_2 \ dx + \left[ \frac{dz_e}{dx} w_1 \right]_0^e + \left[ \frac{dz_e}{dx} W_2 \right]_0^e + B_i
\]

\[
\frac{\Delta H_I L_{eb}}{E_b A_b} + \alpha \Delta T_b L_{et} = -\int_0^e \left[ \frac{d^2 z_b}{dx^2} \right] w_1 \ dx - \int_0^e \left[ \frac{d^2 z_b}{dx^2} \right] W_2 \ dx + \left[ \frac{dz_b}{dx} w_1 \right]_0^e + \left[ \frac{dz_b}{dx} W_2 \right]_0^e + B_b
\]

where \(w_1 \in (0, e)\), \(w_2 \in (e, l)\). The terms \(\left[ \frac{dz}{dx} w_1 \right]_0^e + \left[ \frac{dz}{dx} W_2 \right]_0^l\) and \(\left[ \frac{dz}{dx} w_1 \right]_0^e + \left[ \frac{dz}{dx} W_2 \right]_0^l\)
Then, substituting Eqs.(3.3.14) (3.2.20) and (3.2.21) into Eq.(3.3.20), it gives

\[
\frac{\Delta H_c L_c}{E_i A_i} + \alpha \Delta T L_{\gamma} = -C(d_i - b_i) + B_i
\]

(3.3.21)

\[
\frac{\Delta H_b L_{\delta}}{E_b A_b} + \alpha \Delta T L_{\delta} = -C(d_b - b_b) + B_b
\]

where

\[
C = \left[ -4 \left( -\frac{2}{l^2} \right) \frac{1}{\Delta H_c + \Delta H_b} \right] \int_0^l \left[ x \left( 1 - \frac{e}{l} \right) - 4 \Delta H_b (d_b - b_b) \frac{x}{l} \left( 1 - \frac{x}{l} \right) - 4 \Delta H_i (d_i - b_i) \frac{x}{l} \right] dx
\]

Through multiplying \((H_{\delta b} + \Delta H_b)\) on both sides of the above equations, performing necessary integration and rewriting them in a more visible way, the two linear coupled cable truss equations can be obtained and they are similar to Eqs.(3.3.12).

### 3.4 Coefficients in the Governing Equations of Coupled Linear Cable Trusses

As mentioned before, the coefficients \(c_{b,j}\) and \(c_{t,j}\) for \(j = 1, 2, 3\) in Eqs.(3.3.12) which depend on the loading types can be obtained by performing suitable necessary integrations.

**Case 1: Cable Truss under Uniformly Distributed Vertical Load**

For the cable truss under a uniformly distributed vertical load applied over the entire span, the coefficients in Eqs.(3.3.12) are given as follows.

For the bottom cable
\[c_{b1} = \left( H_{ob} + H_{ot} \right) + E_b \frac{A_b}{L_{ob}} \left[ \frac{16}{3l}(d_b - b_b)^2 \right] \]

\[c_{b2} = E_b \frac{A_b}{L_{ob}} \left[ \frac{16}{3l} (d_b - b_b)(d_i - b_i) \right] \]

\[c_{b3} = E_b \frac{A_b}{L_{ob}} \left[ \alpha T_i L_{rb} \left( H_{ob} + H_{ot} \right) - \left( \frac{2}{3} \right)(d_b - b_b)ql + B_b \left( H_{ob} + H_{ot} \right) \right] \]

For the top cable

\[c_{t1} = E_t \frac{A_t}{L_{et}} \left[ \frac{16}{3l} (d_b - b_b)(d_i - b_i) \right] \]

\[c_{t2} = \left( H_{ob} + H_{ot} \right) + E_t \frac{A_t}{L_{et}} \left[ \frac{16}{3l} (d_i - b_i)^2 \right] \]

\[c_{t3} = E_t \frac{A_t}{L_{et}} \left[ \alpha T_i L_{rt} \left( H_{ob} + H_{ot} \right) - \left( \frac{2}{3} \right)(d_i - b_i)ql + B_t \left( H_{ob} + H_{ot} \right) \right] \]

**Case 2: Cable Truss under Uniformly Distributed Vertical Load from \( x=a \) to \( x=b \)**

For the cable truss under a uniformly distributed vertical load applied from \( x = a \) to \( x = b \) along the span, the coefficients \( c_{b1}, c_{b2} \) and \( c_{t1}, c_{t2} \) are same as those in case 1, but only \( c_{b3} \) in the bottom cable equation and \( c_{t3} \) in the top cable equation are different from those in Eqs.(3.3.12) and they are given as follows.

For the bottom cable

\[c_{b3} = E_b \frac{A_b}{L_{ob}} \left[ \alpha T_i L_{rb} \left( H_{ob} + H_{ot} \right) \right] \left( d_b - b_b \right) \left( \frac{4qb^3}{3l^2} - \frac{4qa^3}{3l^2} - \frac{2gb^2}{l} + \frac{2qa^2}{l} \right) + \]

\[B_b \left( H_{ob} + H_{ot} \right) \]

For the top cable

\[c_{t3} = E_t \frac{A_t}{L_{et}} \left[ \alpha T_i L_{rt} \left( H_{ob} + H_{ot} \right) \right] \left( d_i - b_i \right) \left( \frac{4qb^3}{3l^2} - \frac{4qa^3}{3l^2} - \frac{2gb^2}{l} + \frac{2qa^2}{l} \right) + \]

\[B_t \left( H_{ob} + H_{ot} \right) \]
Case 3: Cable Truss under Uniformly Distributed Vertical Load over Left Half Span

For the cable truss under a uniformly distributed vertical load applied over the left half span, only $c_{b3}$ in the bottom cable equation and $c_{t3}$ in the top cable equation are different from those in Eqs.(3.3.12) and they are given as follows.

For the bottom cable

$$c_{b3} = E_b \frac{A_b}{L_{ob}} \left[ \alpha T_{b} L_{b} \left( H_{ob} + H_{ot} \right) - (d_b - b_b) \left( \frac{1}{3} q_l \right) + B_b \left( H_{ob} + H_{ot} \right) \right]$$

For the top cable

$$c_{t3} = E_t \frac{A_t}{L_{ot}} \left[ \alpha T_{t} L_{t} \left( H_{ob} + H_{ot} \right) - (d_t - b_t) \left( \frac{1}{3} q_l \right) + B_t \left( H_{ob} + H_{ot} \right) \right]$$

Case 4: Cable Truss under Vertical Point Load at $x=e$

For the cable truss under a vertical point load applied at $x = e$ along the span, only $c_{b3}$ in the bottom cable equation and $c_{t3}$ in the top cable equation are different from those in Eqs.(3.3.12) and they are given as follows.

For the bottom cable

$$c_{b3} = E_b \frac{A_b}{L_{ob}} \left[ \alpha T_{b} L_{b} \left( H_{ob} + H_{ot} \right) + 4P \left( \frac{e^2}{L^2} - \frac{e}{L} \right) (d_b - b_b) + B_b \left( H_{ob} + H_{ot} \right) \right]$$

For the top cable

$$c_{t3} = E_t \frac{A_t}{L_{ot}} \left[ \alpha T_{t} L_{t} \left( H_{ob} + H_{ot} \right) + 4P \left( \frac{e^2}{L^2} - \frac{e}{L} \right) (d_t - b_t) + B_t \left( H_{ob} + H_{ot} \right) \right]$$
3.5 Solution of Coupled Linear Cubic Cable Truss Algebraic Equations

Gaussian elimination method is applied to solve the algebraic equations similar to Eqs.(3.3.12).

All the derived equations and results for biconvex cable truss can be equally applicable to biconcave cable truss.

For biconvex cable truss, \( (d_b - b_b), (d_t - b_t), \Delta H_b \) and \( \Delta H_t \) are all positive. The final bottom and top horizontal components of cable forces are

\[
H_b = H_{ob} + \Delta H_b
\]
\[
H_t = H_{ot} - \Delta H_t
\]

While, for biconcave cable truss, \( (d_b - b_b), (d_t - b_t), \Delta H_b \) and \( \Delta H_t \) are all negative. In this case, the horizontal components of cable forces are

\[
H_b = H_{ob} - \Delta H_b
\]
\[
H_t = H_{ot} + \Delta H_t
\]

The brief C++ program procedures for linear analysis of the cable truss are stated as following.

Step 1. Input the material and geometric properties of the cable truss structure, ie. load type (1 for uniformly distributed vertical load applied over entire span, 2 for uniformly distributed vertical load applied on partial span, 3 for uniformly distributed vertical load applied over left half span and 4 for vertical point load), \( H_{ob}, H_{ot}, E_b, E_t, A_b, A_t, l, d_b, b_b, d_t, b_t, q, \alpha, \Delta T_b, \Delta T_t, B_b, B_t \), shape type of the cable truss (1 for biconcave cable truss and 2 for biconvex cable truss).
Step 2. Calculate $L_{eb}$, $L_{et}$, $L_{Tb}$, $L_{Tt}$.

Step 3. According to the load type, use corresponding formulas of coefficients to get $c_{bj}$ and $c_{tj}$ for $j = 1, 2, 3$ in coupled linear cable truss equations.

Step 4. Utilize Gaussian elimination method to obtain the additional horizontal components of the cable forces in top and bottom chords, $\Delta H_b$, $\Delta H_t$.

Step 5. Calculate the final bottom and top horizontal components of cable forces after deformation, $H_b$ and $H_t$ corresponding to the type of the cable truss structure.

Step 6. Use different $x$ value to get the moment $M$, and calculate the corresponding deflection $w$.

Step 7. Output $\Delta H_b$, $\Delta H_t$, $H_b$, $H_t$, $M$ and $w$. 
Chapter 4 Nonlinear Theory of Cable Trusses

4.1 Vertical Deflections of Nonlinear Cable Trusses under Different Static Loads

With nonlinear cable truss theory, solution procedures similar to the previous can be used. In nonlinear structural analysis, the terms \( \Delta H_b \frac{dw}{dx} \) and \( \Delta H_t \frac{dw}{dx} \) need to be included and they can lead to the third order terms in \( w \). Integrating both sides of nonlinear differential Eq.(3.2.3) gives

\[
w = \frac{1}{H_{ob} + \Delta H_b + H_{ot} - \Delta H_t} \left[ M - 4\Delta H_b (d_b - b_b) \frac{x}{l} \left( 1 - \frac{x}{l} \right) - 4\Delta H_t (d_t - b_t) \frac{x}{l} \left( 1 - \frac{x}{l} \right) \right]
\]

(4.1.1)

Case 1: Cable Truss under Uniformly Distributed Vertical Load

For the cable truss under a uniformly distributed vertical load \( q \) applied over the entire span, see Figure 3.2(a), substitution Eq.(3.2.6) into Eq.(4.1.1) gives the vertical deflection as

\[
w = \frac{1}{H_{ob} + \Delta H_b + H_{ot} - \Delta H_t} \left[ \frac{ql}{2} \left( x - \frac{x^2}{l} \right) - 4\Delta H_b (d_b - b_b) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) - 4\Delta H_t (d_t - b_t) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) \right]
\]

(4.1.2)

Case 2: Cable Truss under Uniformly Distributed Vertical Load from \( x=a \) to \( x=b \)

For the cable truss under a uniformly distributed vertical load \( q \) applied from \( x = a \) to \( x = b \) along the span, see Figure 3.2(b), substitution Eqs.(3.2.8), (3.2.9) and (3.2.10) into Eq.(4.1.1) gives the vertical deflection as
\[
\begin{align*}
    w &= \frac{1}{H_{ob} + \Delta H_b + H_{ot} - \Delta H_t} \left[ \frac{q_x}{2l} \left( b - a \right)^2 + \frac{q_x}{l} \left( b - a \right) \left( l - b \right) - 4\Delta H_b \right] \\
    & \quad \times \left( d_b - b_h \right) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) - 4\Delta H_t \left( d_i - b_i \right) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) \\
\end{align*}
\]

(4.1.3)

for \( x \in (0, a) \),

\[
\begin{align*}
    w &= \frac{1}{H_{ob} + \Delta H_b + H_{ot} - \Delta H_t} \left[ \frac{q_x}{2l} \left( b - a \right)^2 + \frac{q_x}{l} \left( b - a \right) \left( l - b \right) \right] \\
    & \quad - \frac{q a^2}{2} - 4\Delta H_b \left( d_b - b_h \right) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) - 4\Delta H_t \left( d_i - b_i \right) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) \\
\end{align*}
\]

(4.1.4)

for \( x \in (a, b) \), and

\[
\begin{align*}
    w &= \frac{1}{H_{ob} + \Delta H_b + H_{ot} - \Delta H_t} \left[ \frac{q_x}{2l} \left( b - a \right)^2 + \frac{q_x}{l} \left( b - a \right) \left( l - b \right) \right] \nonumber \\
    & \quad + \frac{q b^2}{2} - 4\Delta H_b \left( d_b - b_h \right) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) - 4\Delta H_t \left( d_i - b_i \right) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) \\
\end{align*}
\]

(4.1.5)

for \( x \in (b, l) \).

**Case 3: Cable Truss under Uniformly Distributed Vertical Load over Left Half Span**

For cable truss under a uniformly distributed vertical load \( q \) applied over the left half of the span from \( x = 0 \) to \( x = l/2 \), see Figure 3.2(c), substitution Eqs.(3.2.14) and (3.2.15) into Eq.(4.1.1) gives the vertical deflection as

\[
\begin{align*}
    w &= \frac{1}{H_{ob} + \Delta H_b + H_{ot} - \Delta H_t} \left[ q_t \left( \frac{3x}{8} - \frac{x^2}{2l} \right) - 4\Delta H_b \left( d_b - b_h \right) \right] \\
    & \quad \times \left( \frac{x}{l} - \frac{x^2}{l^2} \right) - 4\Delta H_t \left( d_i - b_i \right) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) \\
\end{align*}
\]

(4.1.6)

for \( x \in (0, l/2) \), and
\[ w = \frac{1}{H_{ob} + \Delta H_b + H_{at} - \Delta H_t} \left[ \frac{ql}{8} (l-x) - 4\Delta H_b (d_b - b_b) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) - 4\Delta H_t (d_t - b_t) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) \right] \]  
\[(4.1.7)\]

for \( x \in (l/2, l) \).

**Case 4: Cable Truss under Vertical Point Load at \( x=e \)**

For the cable truss under a point load \( P \) which is applied at the location \( x = e \) along the span, see Figure 3.2(d), substitution Eqs.(3.2.18) and (3.2.19) into Eq.(4.1.1) gives the vertical deflection as

\[ w = \frac{1}{H_{ob} + \Delta H_b + H_{at} - \Delta H_t} \left[ Px \left( 1 - \frac{e}{l} \right) - 4\Delta H_b (d_b - b_b) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) - 4\Delta H_t (d_t - b_t) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) \right] \]  
\[(4.1.8)\]

for \( x \in (0, e) \) and

\[ w = \frac{1}{H_{ob} + \Delta H_b + H_{at} - \Delta H_t} \left[ Pe \left( 1 - \frac{x}{l} \right) - 4\Delta H_b (d_b - b_b) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) - 4\Delta H_t (d_t - b_t) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) \right] \]  
\[(4.1.9)\]

for \( x \in (e, l) \).
4.2 Governing Equations for the Nonlinear Cubic Cable Trusses under Four Different Static Loading Cases

Use the similar procedures as shown in Section 3.3.

With nonlinear cable truss theory, the term $\frac{1}{2} \left( \frac{dw}{dx} \right)^2$ in Eq.(3.3.3) can’t be ignored as it would cause the third order term in $w$.

$$\bar{s}_b - s_b = \int_0^l \left[ \frac{du_b}{dx} + \frac{dw}{dx} \frac{dz_b}{dx} + \frac{1}{2} \left( \frac{dw}{dx} \right)^2 \right] dx \quad (4.2.1)$$

Using Hooke’s law and substituting Eq.(3.3.4) into Eq.(4.2.1) gives

$$\int \frac{\Delta H_b}{E_b A_b} \frac{ds_b}{dx} ds_b = \int_0^l \left[ \frac{du_b}{dx} + \frac{dw}{dx} \frac{dz_b}{dx} + \frac{1}{2} \left( \frac{dw}{dx} \right)^2 \right] dx \quad (4.2.2)$$

The left-hand side part of Eq.(4.2.2) can be written as

$$\int \frac{\Delta H_b}{E_b A_b} \left( \frac{ds_b}{dx} \right)^2 dx = \int_0^l \left[ \frac{du_b}{dx} + \frac{dw}{dx} \frac{dz_b}{dx} + \frac{1}{2} \left( \frac{dw}{dx} \right)^2 \right] dx \quad (4.2.3)$$

Similarly for the top chord,

$$\int \frac{\Delta H_t}{E_t A_t} \left( \frac{ds_t}{dx} \right)^2 dx = \int_0^l \left[ \frac{du_t}{dx} + \frac{dw}{dx} \frac{dz_t}{dx} - \frac{1}{2} \left( \frac{dw}{dx} \right)^2 \right] dx \quad (4.2.4)$$

Then the following equations are formulated.

$$\frac{\Delta H_t}{E_t A_t} L_{et} = u_t(l) - u_t(0) + \int_0^l \left( \frac{dw}{dx} \left( \frac{dz_t}{dx} \right) \right) dx - \frac{1}{2} \int_0^l \left( \frac{dw}{dx} \right)^2 dx \quad (4.2.5)$$

$$\frac{\Delta H_b}{E_b A_b} L_{eb} = u_b(l) - u_b(0) + \int_0^l \left( \frac{dw}{dx} \frac{dz_b}{dx} \right) dx + \frac{1}{2} \int_0^l \left( \frac{dw}{dx} \right)^2 dx$$

The coefficients $L_{eb}$, $L_{et}$ are given by Eq.(3.3.8).
Considering the effect of uniform temperature change, substitution Eq. (3.3.9) into Eq. (4.2.1) gives

\[
\frac{\Delta H}{E_A} \frac{L_c}{\alpha} + \alpha \Delta T L_{T_l} = u_t(l) - u_t(0) + \int_0^l \left( \frac{dw}{dx} \frac{dz}{dx} \right) dx - \frac{1}{2} \int_0^l \left( \frac{dw}{dx} \right)^2 dx
\]

\[
\frac{\Delta H_b}{E_b} \frac{L_{T_b}}{\alpha} + \alpha \Delta T L_{T_b} = u_b(l) - u_b(0) + \int_0^l \left( \frac{dw}{dx} \frac{dz}{dx} \right) dx + \frac{1}{2} \int_0^l \left( \frac{dw}{dx} \right)^2 dx
\]

(4.2.6)

where the coefficients \( L_{T_b}, L_{T_l} \) are given by Eq. (3.3.11).

After making appropriate substitution and performing the necessary integration, the coupled system of cubic cable equations (4.2.6) for \( \Delta H_b \) and \( \Delta H_t \) formulated are showed in Eqs. (4.2.7).

\[
\Delta H_b^3 + c_{b1} \Delta H_b^2 + c_{b2} \Delta H_b + c_{b3} \Delta H_b \Delta H_t + c_{b4} \Delta H_b \Delta H_t^2 + c_{b5} \Delta H_b \Delta H_t + c_{b6} \Delta H_b \Delta H_t + \\
+ c_{b7} \Delta H_t + c_{b8} \Delta H_t + c_{b9} = 0
\]

(4.2.7)

\[
\Delta H_t^3 + c_{t1} \Delta H_t^2 + c_{t2} \Delta H_t + c_{t3} \Delta H_t \Delta H_b + c_{t4} \Delta H_b \Delta H_t^2 + c_{t5} \Delta H_b \Delta H_t + \\
+ c_{t6} \Delta H_b + c_{t7} \Delta H_t + c_{t8} = 0
\]
Case 1: Cable Truss under Uniformly Distributed Vertical Load

For the cable truss under a uniformly distributed vertical load $q$ applied over the entire span, taking integration by parts for Eqs.(4.2.6) gives

$$
\frac{\Delta H_{i}L_{ai}}{E_{i}A_{i}} + \alpha \Delta T_{i}L_{ni} = - \int_{0}^{l} \left( \frac{d^{2}z_{i}}{dx^{2}} w \right) dx + \frac{1}{2} \int_{0}^{l} \frac{d^{2}w}{dx^{2}}wdx - \frac{1}{2} \left[ \frac{dw}{dx} \right]^{l}_{0} + \left[ \frac{dz_{i}}{dx} w \right]^{l}_{0} + B_{i}
$$

(4.2.8)

where the terms $\frac{1}{2} \left[ \frac{dw}{dx} \right]^{l}_{0} + \left[ \frac{dz_{i}}{dx} w \right]^{l}_{0}$ and $\frac{1}{2} \left[ \frac{dw}{dx} \right]^{l}_{0} + \left[ \frac{dz_{b}}{dx} w \right]^{l}_{0}$ equal zero.

Taking the second derivative of $w$ expressed by (Eq.(4.1.2)) with respect to $x$, it gives

$$
\frac{d^{2}w}{dx^{2}} = \frac{1}{H_{ob} + \Delta H_{b} + H_{ot} - \Delta H_{i}} \left[ \frac{ql}{2} - \Delta H_{b} \left( d_{b} - b_{b} \right) \left( - \frac{2x}{l^{2}} \right) - 4\Delta H_{i} \left( d_{b} - b_{b} \right) \left( - \frac{2x}{l^{2}} \right) \right]
$$

(4.2.9)

Then, substitution Eqs.(3.3.14), (4.1.2) and (4.2.9) into Eq.(4.2.8) gives

$$
\frac{\Delta H_{i}L_{ai}}{E_{i}A_{i}} + \alpha \Delta T_{i}L_{ni} = A \left( d_{i} - b_{i} \right) + B + B_{i}
$$

(4.2.10)

$$
\frac{\Delta H_{b}L_{ob}}{E_{b}A_{b}} + \alpha \Delta T_{b}L_{nb} = A \left( d_{b} - b_{b} \right) - B + B_{b}
$$
where

\[ A = \left[ -4 \left( -\frac{2}{l^2} \right) \frac{1}{H_{ob} + \Delta H_{b} + H_{ot} - \Delta H_{t}} \right] \int_{0}^{l} \frac{q}{2} \left( \frac{x^2}{l} - 4\Delta H_{b} (d_{b} - b_{b}) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) \right) dx \]

\[ B = \left[ \frac{1}{2} \left( \frac{1}{H_{ob} + \Delta H_{b} + H_{ot} - \Delta H_{t}} \right)^2 \right] \int_{0}^{l} \frac{q}{2} \left( \frac{x^2}{l} - 4\Delta H_{b} (d_{b} - b_{b}) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) \right) dx \]

Through multiplying \((H_{ob} + \Delta H_{b} + H_{ot} - \Delta H_{t})^2\) on both sides of the above equations, performing necessary integration and rewriting them in a more visible way, two non-linear coupled cable truss equations similar to Eqs.(4.2.7) are obtained.
Case 2: Cable Truss under Uniformly Distributed Vertical Load from x=a to x=b

For the cable truss under a uniformly distributed vertical load \( q \) applied from \( x = a \) to \( b \) along the span of the truss cable, taking integration by parts for Eqs.(4.2.6) gives

\[
\frac{\Delta H_t L_{ew}^*}{E_t I_t} + \alpha \Delta T_t L_{rt} = - \frac{1}{2} \int_a^b \left( \frac{d^2 z_a}{dx^2} w_1 \right) dx - \int_a^b \left( \frac{d^2 z_b}{dx^2} w_2 \right) dx - \int_b^l \left( \frac{d^2 z_l}{dx^2} w_3 \right) dx + \frac{1}{2} \int_a^b \frac{d^2 w_1}{dx^2} w_1 dx + \frac{1}{2} \int_a^b \frac{d^2 w_2}{dx^2} w_2 dx + \frac{1}{2} \int_b^l \frac{d^2 w_3}{dx^2} w_3 dx -
\]

\[
\frac{1}{2} \left[ \frac{d z_a}{dx} w_1 \right]_a^b + \frac{1}{2} \left[ \frac{d z_b}{dx} w_2 \right]_a^b - \frac{1}{2} \left[ \frac{d z_b}{dx} w_3 \right]_a^b + \frac{1}{2} \frac{dz}{dx} w_1 \bigg|_a^0 + \frac{1}{2} \frac{dz}{dx} w_2 \bigg|_a^b + \frac{1}{2} \frac{dz}{dx} w_3 \bigg|_a^l+
\]

\[
\frac{1}{2} \left[ \frac{d z_a}{dx} w_1 \right]_a^b + \frac{1}{2} \left[ \frac{d z_b}{dx} w_2 \right]_a^b - \frac{1}{2} \left[ \frac{d z_b}{dx} w_3 \right]_a^b + \frac{1}{2} \frac{dz}{dx} w_1 \bigg|_a^0 + \frac{1}{2} \frac{dz}{dx} w_2 \bigg|_a^b + \frac{1}{2} \frac{dz}{dx} w_3 \bigg|_a^l+
\]

(4.2.11)

where \( w_1 \in (0, c) \), \( w_2 \in (c, d) \) and \( w_3 \in (d, l) \). The terms \(- \frac{1}{2} \left[ \frac{d w_1}{dx} w_1 \right]_0^a - \frac{1}{2} \left[ \frac{d w_1}{dx} w_1 \right]_a^b - \frac{1}{2} \left[ \frac{d w_3}{dx} w_3 \right]_b^l - \frac{1}{2} \left[ \frac{d w_3}{dx} w_3 \right]_b^l\) equal zero.

Taking the second derivative of \( w \) expressed by Eqs.(4.1.3), (4.1.4) and (4.1.5) with respect to \( x \), it gives
\[
\frac{d^2w}{dx^2} = \frac{1}{H_{ab} + \Delta H_b + H_{at} - \Delta H_t} \left[ -4\Delta H_b (d_b - b_b) \left( \frac{2x}{l^2} \right) - \right. \\
4\Delta H_t (d_t - b_t) \left( -\frac{2x}{l^2} \right) \left. \right] 
\] (4.2.12)

for \( x \in (0, a) \) and \( x \in (b, l) \),

\[
\frac{d^2w}{dx^2} = \frac{1}{H_{ab} + \Delta H_b + H_{at} - \Delta H_t} \left[ -q - 4\Delta H_b (d_b - b_b) \left( \frac{2x}{l^2} \right) - \right. \\
4\Delta H_t (d_t - b_t) \left( -\frac{2x}{l^2} \right) \left. \right] 
\] (4.2.13)

for \( x \in (a, b) \).

Then, substitution Eqs.(3.3.14), (4.2.12), (4.2.13), (4.1.3), (4.1.4) and (4.1.5) into Eq.(4.2.11) gives

\[
\frac{\Delta H_b L_{ab}}{E_s A_s} + \alpha \Delta T_{L_{T_b}} = A(d_t - b_t) + B + B_t, \\
\frac{\Delta H_b L_{ab}}{E_s A_s} + \alpha \Delta T_{L_{T_b}} = A(d_t - b_t) - B + B_t, 
\] (4.2.14)

where

\[
A = \left[ -4 \left( \frac{2}{l^2} \right) H_{ab} + \Delta H_b + H_{at} - \Delta H_t \right] \left\{ \int_0^a \left[ \frac{qX}{2l} (b-a)^2 + \frac{qX}{l} (b-a)(l-b) - 4\Delta H_b \right. \\
\left. (d_b - b_b) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) - 4\Delta H_t (d_t - b_t) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) \right] dx + \int_a^b \left[ \frac{qX}{2l} (b-a)^2 + \frac{qX}{l} (b-a)(l-b) - 4\Delta H_b \right. \\
\left. (d_b - b_b) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) - 4\Delta H_t (d_t - b_t) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) \right] dx \right\} 
\]
$$B = \left[ \frac{1}{2} \left( H_{ob} + \Delta H_b + H_{ot} - \Delta H_t \right) \right]^2 \left( \int_0^a \frac{qx}{2l} (b-a)^2 + \frac{qx}{l} (b-a)(l-b) - 4\Delta H_b (d_b - b_b) \right)$$

$$\left( \frac{x}{l} - \frac{x^2}{l^2} \right) - 4\Delta H_i (d_i - b_i) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) \left[ -4\Delta H_b (d_b - b_b) \left( -\frac{2}{l^2} \right) \right]$$

$$\left[ -\frac{2}{l^2} \right] dx + \int_a^b \frac{qx}{2l} (b-a)^2 + \frac{qx}{l} (b-a)(l-b) - \frac{qx^2}{2} + qax - \frac{qa^2}{2} + \frac{qx}{l} (b-a)(l-b) -$$

$$4\Delta H_b (d_b - b_b) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) - 4\Delta H_i (d_i - b_i) \left( \frac{x}{l} - \frac{x^2}{l^2} \right) \left[ -q - 4\Delta H_b (d_b - b_b) \left( -\frac{2x}{l^2} \right) \right]$$

Through multiplying \((H_{ob} + \Delta H_b + H_{ot} - \Delta H_t)^2\) on both sides of the above equations, performing necessary integration and rewriting them in a more visible way, two non-linear coupled cable truss equations similar to Eqs.(4.2.7) are obtained.
Case 3: Cable Truss under Uniformly Distributed Vertical Load over Left Half Span

For the cable truss under a uniformly distributed vertical load $q$ applied over the left half of the span from $x = 0$ to $x = l/2$, taking integration by parts for Eqs.(4.2.6) gives

$$
\frac{\Delta H_s L_{cb}}{E_s A_s} + \alpha \Delta T_s L_{rt} = -\frac{l}{2} \left( \int \frac{d^2 z}{dx^2} w_1 \right) dx - \frac{l}{2} \left( \int \frac{d^2 z}{dx^2} w_2 \right) dx + \frac{l}{2} \int_0^l \frac{d^2 w_1}{dx^2} w_1 dx
$$

$$
+ \frac{l}{2} \int_0^l \frac{d^2 w_2}{dx^2} w_2 dx - \frac{l}{2} \int_0^l \frac{d w_1}{dx} w_1 dx - \frac{l}{2} \int_0^l \frac{d w_2}{dx} w_2 dx
$$

$$
= \int_0^l \left( \frac{dz}{dx} w_1 \right) dx + \int_0^l \left( \frac{dz}{dx} w_2 \right) dx
$$

$$
= \int_0^l \left( \frac{dz}{dx} w_1 \right) dx + \int_0^l \left( \frac{dz}{dx} w_2 \right) dx
$$

(4.2.15)

where $w_1 \in (0, l/2)$, $w_2 \in (l/2, l)$. The terms $-\frac{l}{2} \left[ \frac{d w_1}{dx} w_1 \right] \frac{l}{2} - \frac{l}{2} \left[ \frac{d w_2}{dx} w_2 \right] \frac{l}{2}$

$$
\left[ \frac{dz}{dx} w_1 \right] \frac{l}{2} + \left[ \frac{dz}{dx} w_2 \right] \frac{l}{2}
$$

and

$$
\left[ \frac{dz}{dx} w_1 \right] \frac{l}{2} + \left[ \frac{dz}{dx} w_2 \right] \frac{l}{2}
$$

equal zero.

Further, taking the second derivative of $w$ expressed by Eq.(4.1.6) and (4.1.7) with respect to $x$, it gives
\[
\frac{d^2w}{dx^2} = \frac{1}{H_{ob} + \Delta H_b + H_{at} - \Delta H_t} \left[-q - 4\Delta H_b (d_b - b_b) \left( -\frac{2}{l^2} \right) \right]
\]
\[4\Delta H_t (d_t - b_t) \left( -\frac{2}{l^2} \right)] \qquad (4.2.16)

for \( x \in (0, l/2), \) and

\[
\frac{d^2w}{dx^2} = \frac{1}{H_{ob} + \Delta H_b + H_{at} - \Delta H_t} \left[-4\Delta H_b (d_b - b_b) \left( -\frac{2}{l^2} \right) \right]
\]
\[4\Delta H_t (d_t - b_t) \left( -\frac{2}{l^2} \right)] \qquad (4.2.17)

for \( x \in (l/2, l). \)

Then, substitution Eqs. (3.3.14), (4.2.16), (4.2.17), (4.1.6) and (4.1.7) into Eq. (4.2.15) gives

\[
\frac{\Delta H_{L_{ca}}}{E_r A_r} + \alpha \Delta T_r L_{T_r} = A (d_r - b_r) + B + B_r \qquad (4.2.18)
\]

\[
\frac{\Delta H_{L_{ab}}}{E_b A_b} + \alpha \Delta T_r L_{T_r} = A (d_r - b_r) - B + B_r
\]

where

\[
A = \left[-4 \left( -\frac{2}{l^2} \right) \frac{1}{H_{ob} + \Delta H_b + H_{at} - \Delta H_t} \right] \int_0^l \left[ q l \left( \frac{3x - x^2}{2l} \right) - 4\Delta H_b (d_b - b_b) \frac{x}{l} \left( 1 - \frac{x}{l} \right) \right]
\]
\[4\Delta H_t (d_t - b_t) \frac{x}{l} \left( 1 - \frac{x}{l} \right)] dx + \int_0^l \left[ q l \left( l - x \right) - 4\Delta H_b (d_b - b_b) \frac{x}{l} \left( 1 - \frac{x}{l} \right) - 4\Delta H_t (d_t - b_t) \frac{x}{l} \left( 1 - \frac{x}{l} \right) \right] dx
\]
\[\frac{x}{l} \left( 1 - \frac{x}{l} \right) \right] dx\]
\[ B = \left[ \frac{1}{2} \left( H_{ob} + \Delta H_b + H_{ot} - \Delta H_t \right) \right]^2 \int_0^l \left\{ q l \left( \frac{3x}{8} - \frac{x^2}{2l} \right) - 4\Delta H_b \left( d_b - b_b \right) \frac{x}{l} \left( 1 - \frac{x}{l} \right) - 4\Delta H_t \left( d_t - b_t \right) \left( -\frac{2}{l^2} \right) \right\} dx + \int_0^l \frac{q l}{8} \left( l - x \right) dx \]

\[ \left( d_t - b_t \right) \frac{x}{l} \left( 1 - \frac{x}{l} \right) \left[-q - 4\Delta H_b \left( d_b - b_b \right) \left( -\frac{2}{l^2} \right) - 4\Delta H_t \left( d_t - b_t \right) \left( -\frac{2}{l^2} \right) \right] dx + \int_0^l \frac{q l}{8} \left( l - x \right) dx \]

\[ -4\Delta H_b \left( d_b - b_b \right) \frac{x}{l} \left( 1 - \frac{x}{l} \right) - 4\Delta H_t \left( d_t - b_t \right) \left( 1 - \frac{x}{l} \right) \left[-4\Delta H_b \left( d_b - b_b \right) \left( -\frac{2}{l^2} \right) - 4\Delta H_t \left( d_t - b_t \right) \left( -\frac{2}{l^2} \right) \right] dx \]

Through multiplying \((H_{ob} + \Delta H_b + H_{ot} - \Delta H_t)^2\) on both sides of the above equations, performing necessary integration and rewriting them in a more visible way, two non-linear coupled cable truss equations similar to Eqs. (4.2.7) are obtained.
Case 4: Cable Truss under Vertical Point Load at $x=e$

For the cable truss under a vertical point load $P$ applied at the location $x = e$ along the span, taking integration by parts for Eqs.(4.2.6) gives

$$\frac{\Delta H_t L_w}{E_i A_i} + \alpha \Delta \tau_t L_{T_b} = -\epsilon \left( \int_0^e \frac{d^2 z_r}{dx^2} w_1 dx - \int_0^e \frac{d^2 z_r}{dx^2} w_2 dx \right) dx + \frac{1}{2} \int_0^e \frac{d^2 w_2}{dx^2} w_1 dx$$

$$+ \frac{1}{2} \int_0^e \frac{d^2 w_2}{dx^2} w_2 dx - \frac{1}{2} \left[ \frac{dw_1}{dx} w_1 \right]_0^e - \frac{1}{2} \left[ \frac{dw_2}{dx} w_2 \right]_0^e$$

$$\left[ \frac{dz_r}{dx} w_1 \right]_0^e + \left[ \frac{dz_r}{dx} w_2 \right]_0^e + B_i$$

$$\frac{\Delta H_t L_{cb}}{E_b A_b} + \alpha \Delta \tau_t L_{T_b} = -\epsilon \left( \int_0^e \frac{d^2 z_b}{dx^2} w_1 dx - \int_0^e \frac{d^2 z_b}{dx^2} w_2 dx \right) dx + \frac{1}{2} \int_0^e \frac{d^2 w_2}{dx^2} w_1 dx$$

$$- \frac{1}{2} \int_0^e \frac{d^2 w_2}{dx^2} w_2 dx + \frac{1}{2} \left[ \frac{dw_1}{dx} w_1 \right]_0^e + \frac{1}{2} \left[ \frac{dw_2}{dx} w_2 \right]_0^e$$

$$\left[ \frac{dz_b}{dx} w_1 \right]_0^e + \left[ \frac{dz_b}{dx} w_2 \right]_0^e + B_b$$

(4.2.19)

where $w_1 \in (0,e)$, $w_2 \in (e,l)$. The terms $\left[ \frac{dz_r}{dx} w_1 \right]_0^e + \left[ \frac{dz_r}{dx} w_2 \right]_0^e$ and $\left[ \frac{dz_b}{dx} w_1 \right]_0^e + \left[ \frac{dz_b}{dx} w_2 \right]_0^e$ equal zero.

Taking the first and second derivative of $w$ expressed by Eq.(4.1.8) and (4.1.9) with respect to $x$, it gives

$$\frac{dw}{dx} = \frac{1}{H_{sb} + \Delta H_b + H_{as} - \Delta H_s} \left[ P \left( 1 - \frac{e}{l} \right) - 4 \Delta H_b \left( d_b - b \right) \left( \frac{1}{l} - \frac{2x}{l^2} \right) - 4 \Delta H_s \left( d_s - b \right) \left( \frac{1}{l} - \frac{2x}{l^2} \right) \right]$$

(4.2.20)

for $x \in (0,e)$. 

43
\[
\frac{dw}{dx} = \frac{1}{H_{ab} + \Delta H_b + H_{et} - \Delta H_t} \left[ Pe\left(\frac{1}{l}\right) - 4\Delta H_b \left(d_b - b_b\right) \left(1 - \frac{2x}{l^2}\right) - 4\Delta H_t \left(d_t - b_t\right) \left(1 - \frac{2x}{l^2}\right) \right]
\]

(4.2.21)

for \( x \in (e, l) \), and

\[
\frac{d^2w}{dx^2} = \frac{1}{H_{ab} + \Delta H_b + H_{et} - \Delta H_t} \left[ -4\Delta H_b \left(d_b - b_b\right) \left(-\frac{2}{l^2}\right) - 4\Delta H_t \left(d_t - b_t\right) \left(-\frac{2}{l^2}\right) \right]
\]

(4.2.22)

for \( x \in (0, l) \).

Then, substitution Eqs. (3.3.14), (4.2.20), (4.2.21), (4.2.22), (4.1.8) and (4.1.9) into Eqs. (4.2.19) gives

\[
\frac{\Delta H_v L_{et}}{E_r A_r} + \alpha \Delta T_v L_{en} = A \left(d_e - b_e\right) + B + B_e,
\]

(4.2.23)

\[
\frac{\Delta H_v L_{et}}{E_b A_b} + \alpha \Delta T_v L_{en} = A \left(d_b - b_b\right) - B + B_b,
\]

where

\[
A = \left[-4\left(-\frac{2}{l^2}\right) \frac{1}{H_{ab} + \Delta H_b + H_{et} - \Delta H_t} \right] \left[ \int_{e}^{l} q \left(1 - \frac{e}{l}\right) - 4\Delta H_b \left(d_b - b_b\right) \left(1 - \frac{x}{l}\right) - 4\Delta H_t \left(d_t - b_t\right) \left(1 - \frac{2x}{l^2}\right) \right]
\]

\[
(4\Delta H_t \left(d_t - b_t\right) \frac{x}{l} \left(1 - \frac{x}{l}\right) \right)dx + \left[ \int_{e}^{l} q \left(1 - \frac{e}{l}\right) - 4\Delta H_b \left(d_b - b_b\right) \frac{x}{l} \left(1 - \frac{x}{l}\right) - 4\Delta H_t \left(d_t - b_t\right) \frac{x}{l} \left(1 - \frac{x}{l}\right) \right]dx
\]
\[ B = \left[ \frac{1}{2} \left( H_{ob} + \Delta H_b + H_{ot} - \Delta H_t \right)^2 \right]^n \int_0^1 \left[ -4 \Delta H_b (d_b - b_b) \left( 1 - \frac{x}{l} \right)^2 \right] \] 

\[ [q x \left( 1 - \frac{e}{l} \right) - 4 \Delta H_b (d_b - b_b) \frac{x}{l} \left( 1 - \frac{x}{l} \right) - 4 \Delta H_t (d_t - b_t) \frac{x}{l} \left( 1 - \frac{x}{l} \right)] dx + \int_1^1 [q e \left( 1 - \frac{x}{l} \right) - 4 \Delta H_b (d_b - b_b) \left( 1 - \frac{2x}{l^2} \right) - 4 \Delta H_t (d_t - b_t) \left( 1 - \frac{x}{l} \right)]^{22} \]

\[ \Delta H_b (d_b - b_b) \frac{x}{l} \left( 1 - \frac{x}{l} \right) - 4 \Delta H_t (d_t - b_t) \frac{x}{l} \left( 1 - \frac{x}{l} \right) \int_0^1 \left[ -4 \Delta H_b (d_b - b_b) \left( 1 - \frac{2x}{l^2} \right) - 4 \Delta H_t (d_t - b_t) \left( 1 - \frac{x}{l} \right) \right] \]

Through multiplying \((H_{ob} + \Delta H_b + H_{ot} - \Delta H_t)^2\) on both sides of the above equations, performing necessary integration and rewriting them in a more visible way, two non-linear coupled cable truss equations similar to Eqs. (4.2.7) are obtained.
4.3 Coefficients in the Governing Equations ofCoupled Cubic Nonlinear Cable Trusses

As mentioned before, the coefficients $c_{b j}$ and $c_{t j}$ for $j = 1, 2, 3, \ldots, 8$ in Eqs.(4.2.7) which depend on the loading types can be obtained by performing suitable necessary integrations.

**Case 1: Cable Truss under Uniformly Distributed Vertical Load**

For the cable truss under a uniformly distributed vertical load applied over the entire span, the coefficients in Eqs.(4.2.7) are given as follows.

For the bottom cable

$$c_{b1} = 2(H_{ob} + H_{at}) + E_b \frac{A_b}{L_{ob}} \left[ \frac{16}{6l} (d_b - b_b)^2 + \alpha T_b L_{Tb} + B_b \right]$$

$$c_{b2} = E_b \frac{A_b}{L_{ob}} \left[ \frac{16}{3l} (d_b - b_b)(d_i - b_i) - \frac{16}{6l} (d_i - b_i)^2 + \alpha T_b L_{Tb} + B_b \right]$$

$$c_{b3} = -2$$

$$c_{b4} = 1$$

$$c_{b5} = -E_b \frac{A_b}{L_{ob}} \left[ \frac{16}{3l} (d_b - b_b)^2 + 2\alpha T_b L_{Tb} + 2B_b \right] - 2(H_{ob} + H_{at})$$

$$c_{b6} = E_b \frac{A_b}{L_{ob}} \left[ \frac{16}{3l} (d_b - b_b)^2 (H_{ob} + H_{at}) + 2\alpha T_b L_{Tb} (H_{ob} + H_{at}) + 2B_b (H_{ob} + H_{at}) \right] +$$

$$(H_{ob} + H_{at})^2$$

$$c_{b7} = E_b \frac{A_b}{L_{ob}} \left[ \frac{16}{3l} (d_b - b_b)(d_i - b_i)(H_{ob} + H_{at}) + \frac{2}{3} ql (d_b - b_b) + \frac{2}{3} ql (d_i - b_i) - 2\alpha T_b L_{Tb} \right]$$

$$(H_{ob} + H_{at}) - 2B_b (H_{ob} + H_{at})$$

$$c_{b8} = E_b \frac{A_b}{L_{ob}} \left[ -\frac{2}{3} ql (d_b - b_b)(H_{ob} + H_{at}) - \frac{q^2 l^3}{24} + \alpha T_b L_{Tb} (H_{ob} + H_{at})^2 + B_b (H_{ob} + H_{at})^2 \right]$$
For the top cable

\[
c_{t1} = E_t \frac{A_t}{L_{et}} \left[ \frac{16}{3l} (d_t - b_t)(d_t - b_t) + \frac{16}{6l} (d_t - b_t)^2 + \alpha T_t L_{tt} + B_t \right]
\]

\[
c_{t2} = -2 (H_{ob} + H_{ot}) + E_t \frac{A_t}{L_{et}} \left[ - \frac{16}{6l} (d_t - b_t)^2 + \alpha T_t L_{tt} + B_t \right]
\]

\[
c_{t3} = 1
\]

\[
c_{t4} = -2
\]

\[
c_{t5} = E_t \frac{A_t}{L_{et}} \left[ \frac{16}{3l} (d_t - b_t)^2 - 2\alpha T_t L_{tt} - 2B_t \right] + 2 (H_{ob} + H_{ot})
\]

\[
c_{t6} = E_t \frac{A_t}{L_{et}} \left[ \frac{16}{3l} (d_b - b_b)(d_t - b_t)(H_{ob} + H_{ot}) - \frac{2}{3} ql (d_t - b_t) - \frac{2}{3} ql \right]
\]

\[
(d_b - b_b) + 2\alpha T_t L_{tt} (H_{ob} + H_{ot}) + 2B_t (H_{ob} + H_{ot})
\]

\[
c_{t7} = (H_{ob} + H_{ot})^2 + E_t \frac{A_t}{L_{et}} \left[ \frac{16}{3l} (d_t - b_t)^2 (H_{ob} + H_{ot}) - 2\alpha T_t L_{tt} (H_{ob} + H_{ot}) - 2B_t
\]

\[
(H_{ob} + H_{ot})
\]

\[
c_{t8} = E_t \frac{A_t}{L_{et}} \left[ - \frac{2}{3} ql (d_t - b_t)(H_{ob} + H_{ot}) + \frac{q^2 l^3}{24} + \alpha T_t L_{tt} (H_{ob} + H_{ot})^2 + B_t (H_{ob} + H_{ot})^2 \right]
\]

**Case 2: Cable Truss under Uniformly Distributed Vertical Load from x=a to x=b**

For the cable truss under a uniformly distributed vertical load applied from \(x = a\) to \(x = b\) along the span, the coefficients \(c_{b1}, c_{b2}, c_{b3}, c_{b4}, c_{b5}, c_{b6}\), and \(c_{t1}, c_{t2}, c_{t3}, c_{t4}, c_{t5}, c_{t7}\) in Eqs.(4.2.7) are same as those in case 1, but \(c_{b7}, c_{b8}\) for bottom cable equation and \(c_{t6}, c_{t8}\) for top cable equation are different which are given as follows.
For the bottom cable

\[
c_{b7} = E_b \frac{A_b}{L_{ob}} \left[ -\frac{qa^2}{l} \left( -\frac{4a}{3l} + 2 \right) \left[ (d_b - b_b) + (d_l - b_l) \right] - \frac{qb^2}{l} \left( \frac{4b}{3l} - 2 \right) \left[ (d_b - b_b) + (d_l - b_l) \right] \right] + \frac{16}{3l} (d_b - b_b) (d_l - b_l) (H_{ob} + H_{ot}) - 2aT_bL_{TB} (H_{ob} + H_{ot}) - 2B_b (H_{ob} + H_{ot}) \]

\[
c_{b8} = E_b \frac{A_b}{L_{ob}} \left[ -\frac{qa^2}{l} \left( -\frac{4a}{3l} + 2 \right) (d_b - b_b) (H_{ob} + H_{ot}) + \frac{qa^2}{l} \left( -\frac{qal}{3} + \frac{qa^2}{8} \right) + \frac{qb^2}{l} \left( \frac{4b}{3l} - 2 \right) \right] + (d_b - b_b) (H_{ob} + H_{ot}) + \frac{qb^2}{l} \left( -\frac{qbl}{6} + \frac{qb^2}{8} \right) + q^2 ab \left( -\frac{ab}{4l} + \frac{a}{2} \right) + 2\alpha T_bL_{TB} (H_{ob} + H_{ot}) \]

\[
B_b \left( H_{ob} + H_{ot} \right)^2 \]

For the top cable

\[
c_{t6} = E_t \frac{A_t}{L_{ot}} \left[ -\frac{qa^2}{l} \left( -\frac{4a}{3l} + 2 \right) \left[ (d_l - b_l) + (d_l - b_l) \right] + \frac{qb^2}{l} \left( \frac{4b}{3l} - 2 \right) \left[ (d_b - b_b) + (d_l - b_l) \right] \right] + \frac{16}{3l} (d_b - b_b) (d_l - b_l) (H_{ob} + H_{ot}) + 2aT_tL_{TB} (H_{ob} + H_{ot}) + 2B_t (H_{ob} + H_{ot}) \]

\[
c_{t8} = E_t \frac{A_t}{L_{ot}} \left[ -\frac{qa^2}{l} \left( -\frac{4a}{3l} + 2 \right) (d_l - b_l) (H_{ob} + H_{ot}) - \frac{qa^2}{l} \left( -\frac{qal}{3} + \frac{qa^2}{8} \right) + \frac{qb^2}{l} \left( \frac{4b}{3l} - 2 \right) \right] + (d_l - b_l) (H_{ob} + H_{ot}) - \frac{qb^2}{l} \left( -\frac{qbl}{6} + \frac{qb^2}{8} \right) - q^2 ab \left( -\frac{ab}{4l} + \frac{a}{2} \right) + 2\alpha T_tL_{TB} (H_{ob} + H_{ot}) \]

\[
B_t \left( H_{ob} + H_{ot} \right)^2 \]

**Case 3: Cable Truss under Uniformly Distributed Vertical Load over Left Half Span**

For the cable truss under a uniformly distributed vertical load applied over the left half span, only \( c_{b7}, c_{b8} \) for bottom cable equation and \( c_{t6}, c_{t8} \) for top cable equation are different from those in Case 1 and they are given as follows.
For the bottom cable

\[
c_{b7} = E_b \frac{A_b}{L_{cb}} \left[ \frac{ql}{3} \left( (d_b - b_b) + (d_i - b_i) \right) + \frac{16}{3l} (d_b - b_b)(d_i - b_i)(H_{ob} + H_{ot}) - 2\alpha T_b L_{tb} \right. \\
\left. \left( H_{ob} + H_{ot} \right) - 2B_b \left( H_{ob} + H_{ot} \right) \right] \\
c_{b8} = E_b \frac{A_b}{L_{cb}} \left[ \frac{ql}{3} \left( d_b - b_b \right) \left( H_{ob} + H_{ot} \right) - \frac{5q^2 l^3}{384} + \alpha T_b L_{tb} \left( H_{ob} + H_{ot} \right)^2 + B_b \left( H_{ob} + H_{ot} \right)^2 \right] 
\]

For the top cable

\[
c_{t6} = E_t \frac{A_t}{L_{ct}} \left[ -\frac{ql}{3} \left( (d_b - b_b) + (d_i - b_i) \right) + \frac{16}{3l} (d_b - b_b)(d_i - b_i)(H_{ob} + H_{ot}) + 2\alpha T_t L_{tt} \right. \\
\left. \left( H_{ob} + H_{ot} \right) + 2B_t \left( H_{ob} + H_{ot} \right) \right] \\
c_{t8} = E_t \frac{A_t}{L_{ct}} \left[ \frac{5q^2 l^3}{384} - \frac{ql}{3} \left( d_i - b_i \right) \left( H_{ob} + H_{ot} \right) + \alpha T_t L_{tt} \left( H_{ob} + H_{ot} \right)^2 + B_t \left( H_{ob} + H_{ot} \right)^2 \right] 
\]

**Case 4: Cable Truss under Vertical Point Load at x=e**

For the cable truss under a vertical point load applied at \( x = e \) along the span, only \( c_{b7}, c_{b7}, c_{b8} \) for bottom cable equation and \( c_{t6}, c_{t7}, c_{t8} \) for top cable equation are different from those in Case 1 and they are given as follows.

For the bottom cable

\[
c_{b7} = E_b \frac{A_b}{L_{cb}} \left[ -\frac{4}{l} \left( d_b - b_b \right) \left( Pg - P e^2 \right) + \frac{4}{l} (d_i - b_i) \left( Pg - P e^2 \right) + \frac{16}{3l} (d_b - b_b)(d_i - b_i) \left( H_{ob} + H_{ot} \right) - 2\alpha T_b L_{tb} \left( H_{ob} + H_{ot} \right) \right. \\
\left. \left( H_{ob} + H_{ot} \right) - 2B_b \left( H_{ob} + H_{ot} \right) \right] \\
c_{b8} = E_b \frac{A_b}{L_{cb}} \left[ -\frac{4}{l} \left( Pg - P e^2 \right) (d_b - b_b) \left( H_{ob} + H_{ot} \right) - \frac{1}{2} P^2 (a - \frac{a^2}{l}) + \alpha T_b L_{tb} \left( H_{ob} + H_{ot} \right)^2 \right. \\
\left. + B_b \left( H_{ob} + H_{ot} \right)^2 \right] 
\]
For the top cable

\[
c_{i6} = \frac{A_i}{L_{ct}} \left( -\frac{4}{l^2} (Pe_i - pe^2_i) (d_i - b_i) - \frac{4}{l^2} (Pe_i - Pe^2_i) (d_b - b_b) + + \frac{16}{3l} (d_b - b_b) (d_t - b_t) \right) \\
(H_{ob} + H_{ot}) + 2\alpha T_l L_{fr} (H_{ob} + H_{ot}) + 2B_i (H_{ob} + H_{ot}) \right]
\]

\[
c_{i8} = \frac{A_i}{L_{ct}} \left( -\frac{4}{l^2} (Pe_i - Pe^2_i) (d_i - b_i) (H_{ob} + H_{ot}) - \frac{1}{2} p^2 \left( a - \frac{a^2}{l} \right) + \alpha T_l L_{fr} (H_{ob} + H_{ot}) \right)^2 + B_i (H_{ob} + H_{ot})^2 \right]
\]

4.4 Solution of Coupled Nonlinear Cubic Cable Truss Algebraic Equations

Newton–Raphson iterations are employed to solve the non-linear cable algebraic equations given by Eqs.(4.2.7).

All the formulated equations and results for biconvex cable truss are equally applicable to biconcave cable truss as mentioned before in section 3.5.

For biconvex cable truss, the resulting bottom and top horizontal components of cable forces can be obtained with Eqs.(3.5.1). However, for biconcave cable truss, the resulting horizontal components of cable forces can be calculated with Eqs.(3.5.2).

Following are the brief C++ program procedures for nonlinear analysis of the cable truss.

Step 1. Input the material and geometric properties of the cable truss structure, ie. load type (1 for uniformly distributed vertical load applied over entire span, 2 for uniformly distributed vertical load applied on partial span, 3 for uniformly distributed vertical load applied over left half span and 4 for vertical point load), \(H_{ob}, H_{ot}, E_b, E_t, A_b, A_t, l, d_b, b_b, d_t, b_t, q, \alpha, \Delta T_b, \Delta T_t, B_b,\)
$B_t$, shape type of the cable truss (1 for biconcave cable truss and 2 for biconvex cable truss), starting values of $\Delta H_b$, $\Delta H_t$.

Step 2. Calculate $L_{eb}$, $L_{et}$, $L_{TB}$, $L_{Tt}$.

Step 3. According to the load type, use corresponding formulas of coefficients to get $c_{bj}$ and $c_{tj}$ for $j = 1, 2…8$ in coupled nonlinear cable truss equations.

Step 4. Utilize Newton–Raphson iterations to obtain the additional horizontal components of the cable forces in top and bottom chords, $\Delta H_b$ and $\Delta H_t$.

Step 5. Calculate the final bottom and top horizontal components of cable forces after deformation, $H_b$ and $H_t$ corresponding to the type of the cable truss structure.

Step 6. Use different $x$ value to get the moment $M$, and calculate the corresponding deflection $w$.

Step 7. Output $\Delta H_b$, $\Delta H_t$, $H_b$, $H_t$, $M$ and $w$. 
Chapter 5 Numerical Analysis of Linear and Nonlinear Cable Trusses

In this chapter, the numerical analysis is performed using Gaussian elimination method and Newton–Raphson iteration method to analyze the linear and nonlinear static cable truss systems respectively, by writing the C++ programs. Through using the same material and geometric properties of the cable truss assumed in Kmet and Kokorudova (2009), very similar final linear and nonlinear horizontal components of the cable forces in both top and bottom chords, as well as the mid-span vertical deflections are obtained by the written C++ programs, compared with the solutions presented in the previous paper. And this slight differences are caused by the different equations are used in this thesis in order to obtain more accurate $L_{et}$ and $L_{eb}$ values. Therefore, it can be reasonable inferred that the written C++ programs can give the correct results.

Here, both linear and nonlinear responses of symmetric and asymmetric cable trusses subjected to four different loads and the relative errors between the results of linear and nonlinear cable trusses are obtained and discussed.

The relative error are calculated by

$$relative\ error = 100\times\left(\frac{linear\ result - nonlinear\ result}{nonlinear\ result}\right)$$

5.1 Different Span-to-Sag Ratios of Symmetric Cable Trusses

In this section, different span-to-sag ratios are considered for symmetric cable trusses under four different loads in numerical analysis.

The geometric and material properties of the cable trusses are given as follows. The
length of the cable truss span is \( l = 60m \). The Young’s moduli of elasticity for both bottom and top cables are \( E_b = E_t = 1.5 \times 10^{11} N/m^2 \). The cross-sectional areas for bottom and top cables are \( A_b = 1.3 \times 10^{-3} m^2 \) and \( A_t = 2.0 \times 10^{-3} m^2 \). For symmetric cable trusses, the initial horizontal pretension components of cable forces in both bottom and top chords are \( H_{ob} = H_{ot} = 600kN \). The span-to-sag ratio of the top cable equals to span-to-camber ratio of the bottom stabilizing cable. Thirty-six different ratios being \( l/s = l/c = 2.5, 5.0, 7.5, 10, 12.5, 15, 17.5, 20, 22.5, 25, 27.5, 30, 32.5, 35, 37.5, 40, 42.5, 45, 47.5, 50, 52.5, 55, 57.5, 60, 62.5, 65, 67.5, 70, 72.5, 75, 77.5, 80, 82.5, 85, 87.5 \) and \( 90 \) are considered in analysis. The geometrical properties of symmetric cable trusses are specified as \( d_b = d_t = 0.5m \) and \( b_b = b_t = 24.5, 12.5, 8.5, 6.5, 5.3, 4.5, 3.93, 3.5, 3.17, 2.9, 2.68, 2.50, 2.35, 2.21, 2.10, 2.00, 1.91, 1.83, 1.76, 1.70, 1.64, 1.59, 1.54, 1.50, 1.46, 1.42, 1.39, 1.36, 1.33, 1.30, 1.27, 1.25, 1.23, 1.21, 1.19 \) and \( 1.17m \) as \( l = l/c \times (b_b - d_b) = l/s \times (b_t - d_t) \).

Horizontal components of cable forces in bottom and top chords as well as the mid-span vertical deflections under applied load versus span-to-sag ratios of the carrying cable truss obtained by the presented coupled cubic non-linear cable truss equations are compared with those obtained by the coupled linear cable truss equations.

5.1.1 Symmetric Cable Truss under Uniformly Distributed Vertical Load q=10kN/m

The geometry of the biconcave symmetric cable truss under a uniformly distributed vertical load applied over entre span is shown in Figure 5.1.
Figure 5.1 Geometry of a Biconcave Symmetric Cable Truss under Uniformly Distributed Vertical Load

Figure 5.2 Horizontal Components of Cable Forces in Top Chord of Symmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios

Figure 5.3 Relative Errors of Ht of Linear Cable Truss vs. Span-to-sag Ratios
As assumed in section 3.1, the cables work only in tension. While according to the results obtained by linear analysis as shown in Figure 5.4, when the span-to-sag ratio is larger than 25 and less than 80, the horizontal components of bottom cable forces are less than 0 and hence it is found that the linear analysis is not effective in this situation.

From Figure 5.2 and Figure 5.4, it is observed that the nonlinear behavior of bottom cable is more obvious than that of top cable. The difference of the horizontal components of cable forces in bottom chord obtained by both linear and nonlinear analyses becomes obvious as the span-to-sag ratio becomes larger than 10. The horizontal components of cable forces in top chord becomes obviously different as the
span-to-sag ratio is larger than 15.

As shown in Figure 5.3, the relative error of linear result of the horizontal tension force component in top chord increases slowly. When the span-to-sag ratios are larger than 80, the relative error is greater than 10%. The largest one is around 12% when the span-to-sag ratio increases to 90.

For the bottom chord, as shown in Figure 5.5, the relative error of Hb keeps greater than 10% when the span-to-sag ratio is larger than 15. The largest one is about 130% which is quite large compared to the relative error of Ht.

![Figure 5.6 Mid-span Vertical Deflections of Symmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios](image)

![Figure 5.7 Relative Errors of Hb of Linear Cable Truss vs. Span-to-sag Ratios](image)

In Figure 5.6, the mid-span vertical deflections of the cable truss obtained by linear
and nonlinear analyses start to deviate when span-to-sag ratio is larger than 15. As shown in Figure 5.7, the relative error of $w$ increases as the span-to-sag ratio becomes larger. When the span-to-sag ratio keeps higher than 25, the relative error is greater than 10%. The largest relative error is around 80% when the span-to-sag ratio increases to 90.

### 5.1.2 Symmetric Cable Truss under Uniformly Distributed Vertical Load $q=10\text{kN/m from } x=20\text{m to } x=40\text{m}$

The geometry of the biconcave symmetric cable truss under a uniformly distributed vertical load applied along the span from $x=20\text{m}$ to $x=40\text{m}$ is shown in Figure 5.8.

![Figure 5.8 Geometry of a Biconcave Symmetric Cable Truss under Uniformly Distributed Vertical Load along Part of the Span](image)

**Figure 5.8** Geometry of a Biconcave Symmetric Cable Truss under Uniformly Distributed Vertical Load along Part of the Span

![Figure 5.9 Horizontal Components of Cable Forces in Top Chord of Symmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios](image)

**Figure 5.9** Horizontal Components of Cable Forces in Top Chord of Symmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios
In Figure 5.9, the horizontal components of cable forces in top chord obtained by nonlinear analysis are always greater than linear results. And as shown in Figure 5.10, the relative error of $H_t$ of linear cable truss becomes greater than 10% when span-to-sag ratio is larger than 75, and the largest relative error reaches 13% when the span-to-sag ratio increases to 90.

Figure 5.10 Relative Errors of $H_b$ of Linear Cable Truss vs. Span-to-sag Ratios

Figure 5.11 Horizontal Components of Cable Forces in Bottom Chord of Symmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios
From Figure 5.11, the horizontal components of cable forces in bottom chord obtained by nonlinear analysis are always greater than linear results. And as shown in Figure 5.12, the relative errors of Hb of linear cable truss are greater than 10% when the span-to-sag ratios are larger than 10. The largest one is around 45% when the span-to-sag ratio increases to 70, and then the relative error becomes stable as span-to-sag ratio continuously increasing.

Figure 5.13 Mid-span Vertical Deflections of Symmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios
In Figure 5.13, both of the mid-span vertical deflections obtained by linear and nonlinear analyses increase as span-to-sag ratio increasing. And the linear results are always larger than the nonlinear results. In addition, as shown in Figure 5.14, the relative errors of w of linear cable truss are greater than 10% when the span-to-sag ratios are larger than 5. And the largest one reaches around 41% when span-to-sag ratio increases to 90.

5.1.3 Symmetric Cable Truss under Uniformly Distributed Vertical Load

$q=10\text{kN/m over Left Half Span}$

The geometry of the biconcave symmetric cable truss under a uniformly distributed vertical load applied over left half span is shown in Figure 5.15.
In Figure 5.16, the horizontal components of cable forces in top chord obtained by linear analysis are always smaller than those obtained by nonlinear analysis. Besides, as shown in Figure 5.17, when the span-to-sag ratios are larger than 70, the relative errors of Ht of linear cable truss become greater than 10%. And the largest relative error is about 15%, when the span-to-sag ratio increases up to 90.
From Figure 5.18, the horizontal components of cable forces in bottom chord obtained by linear analysis are always less than those obtained by nonlinear analysis. Besides, as shown in Figure 5.19, when the span-to-sag ratios are larger than 2.5, relative errors of Hb of linear cable truss become greater than 10%. And the largest one is about 50% as span-to-sag ratio increasing to 60. Then the relative error becomes stable eventually.
In Figure 5.20, both of the mid-span vertical deflections obtained by linear and nonlinear analyses increase as span-to-sag ratio increasing. And the linear results are always larger than the nonlinear results. In addition, as shown in Figure 5.21, the relative error reaches 45% which is significant when span-to-sag ratio increases up to 90.
5.1.4 Symmetric Cable Truss under Vertical Point Load $P=200$ kN at $x=30$m

The geometry of the biconcave symmetric cable truss under a vertical point load applied in the middle of the cable span is shown in Figure 5.22.

![Figure 5.22 Geometry of a Biconcave Symmetric Cable Truss under Vertical Point Load](image)

From Figure 5.23, the horizontal components of cable forces in top chord obtained by nonlinear analysis are always greater than linear results. Besides, as shown in Figure

![Figure 5.23 Horizontal Components of Cable Forces in Top Chord of Symmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios](image)

![Figure 5.24 Relative Errors of Ht of Linear Cable Truss vs. Span-to-sag Ratios](image)
5.24, almost all relative errors of Ht of linear cable truss are higher than 10% as span-to-sag ratio changes. The largest relative error reaches around 15% when the span-to-sag ratio increases up to 90.

In Figure 5.25, the horizontal components of cable forces in top chord obtained by nonlinear analysis are always higher than linear results. In addition, as shown in Figure 5.26, all relative errors of Hb of linear cable truss are higher than 10% as span-to-sag ratio changes. And the relative error first reaches about 52% as span-to-sag ratio increasing to 60, and then it becomes stable as span-to-sag ratio continuously increasing.
In Figure 5.27, the mid-span vertical deflections obtained by linear analysis are always larger than the nonlinear results, and they both increase when span-to-sag ratio becomes larger. Besides, as shown in Figure 5.28, all relative errors of w of linear cable truss are higher than 10% as span-to-sag ratio changes. And the relative error tends to be larger as span-to-sag ratio increasing. The largest relative error is around 48%, when the span-to-sag ratio increases up to 90.

From the above numerical analyses in Section 5.1, it can be summarized as follows.

1. For the symmetric cable truss under the entire uniformly distributed vertical load, the horizontal components of bottom cable forces obtained by linear analysis
become negative when the span-to-sag ratio is larger than 25 and less than 80, which means the linear analysis not works when the span-to-sag ratio falls in this range. On the other hand, horizontal components of bottom cable forces obtained by the nonlinear analysis are still greater than 0. It is seen that there are some limitations with linear analysis. Therefore, in those cases, only nonlinear cable truss theory is applicable.

2. Through the comparisons between the horizontal components of the bottom and top cable forces, it is observed that the deviation between the horizontal components of the bottom cable forces obtained by the linear and nonlinear analyses is larger than that of top cable forces. Thus, the nonlinear behavior of the bottom cable is more obvious than that of the top cable. In this case, the nonlinear cable truss theory is necessary to be used in order to get more accurate results.

3. It is found that the horizontal components of the tension forces in both top and bottom chords obtained by linear analysis are always smaller than those obtained by nonlinear analysis as span-to-sag ratio changes, whereas the mid-span vertical deflections obtained by linear analysis are greater than nonlinear results. Thus, it can be inferred that the stiffness of the nonlinear cable truss is higher than that of linear truss. Using linear cable truss theory to calculate the deflection of the cable truss is considerably safer and more conservative.

4. Regard to the horizontal components of the cable forces in bottom chord for symmetric cable trusses under various loads, the relative errors of linear results are around 15% as span-to-sag ratio increases up to 90. For the horizontal tension force components in top chord, the largest errors of linear results for each case are all greater than 40% as span-to-sag ratio changes. Further, regard to the mid-span
vertical deflections obtained by both linear and nonlinear analyses, the largest relative errors for each case are higher than 40% and some even up to 80%. Therefore, the nonlinear cable truss theory is necessary to be adopted in order to get a relatively accurate result.
5.2 Different Span to Sag Ratios of Asymmetric Cable Trusses

In this section, different span-to-sag ratios are considered for analyzing the asymmetric cable trusses numerically under four different loads. The geometric and material properties of the cable truss are given as follows. The length of the cable truss span is \( l = 60m \). The Young’s moduli of elasticity of both cables are \( E_b = E_t = 1.5 \times 10^{11}N/m^2 \). The cross-sectional areas are \( A_b = 1.3 \times 10^{-3} m^2 \) and \( A_t = 2.0 \times 10^{-3} m^2 \).

For asymmetric cable trusses, thirty-six different span-to-sag ratios of the top cable, \( l/s = 2.5, 5, 7.5, 10, 12.5, 15, 17.5, 20, 22.5, 25, 27.5, 30, 32.5, 35, 37.5, 40, 42.5, 45, 47.5, 50, 52.5, 55, 57.5, 60, 62.5, 65, 67.5, 70, 72.5, 75, 77.5, 80, 82.5, 85, 87.5 and 90 \) are considered in numerical analysis. The span-to-camber ratio keeps constantly as \( l/c = 25 \). With \( l = l/c \times (b_b - d_b) = l/s \times (b_t - d_t) \), the following data for the geometrical properties of cable trusses are considered: \( b_b = 2.9m, b_t = 24.5, 12.5, 8.5, 6.5, 5.3, 4.5, 3.93, 3.5, 3.17, 2.9, 2.68, 2.50, 2.35, 2.21, 2.10, 2.00, 1.91, 1.83, 1.76, 1.70, 1.64, 1.59, 1.54, 1.50, 1.46, 1.42, 1.39, 1.36, 1.33, 1.30, 1.27, 1.25, 1.23, 1.21, 1.19 \) and \( 1.17m, d_b = d_t = 0.5m \). The initial horizontal component of pretension cable force in the bottom chord is \( H_{ob} = 600kN \). In the top chord, initial horizontal components of pretension cable forces are calculated with corresponding various sag values of the carrying top cable from the equation of internal equilibrium for the pre-stressed unloaded asymmetric cable truss as \( H_{ot} = H_{ob}c \). The corresponding \( H_{ot} \) values are 60, 120, 180, 240, 300, 360, 420, 480, 540, 600, 660, 720, 780, 840, 900, 960, 1020, 1080, 1140 and 1200kN. The horizontal components of cable forces in the bottom and top chords as well as the mid-span vertical deflections under applied loads versus the span-to-sag ratios of the top chord obtained by the present coupled cubic non-linear cable truss equations are analyzed and compared with those obtained
by the coupled linear cable truss equations.

5.2.1 Asymmetric Cable Truss under Uniformly Distributed Vertical Load

$q = 10 \text{kN/m}$

The geometry of the biconcave asymmetric cable truss under a uniformly distributed vertical load applied over the span is shown in Figure 5.29.

Figure 5.29 Geometry of a Biconcave Asymmetric Cable Truss under Uniformly Distributed Vertical load

Figure 5.30 Horizontal Components of Cable Forces in Top Chord of Asymmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios
In Figure 5.30, the horizontal components of cable forces in top chord obtained by both linear and nonlinear analyses tend to be larger as span-to-sag ratio increasing. As shown in Figure 3.1, the error for Ht of linear cable truss becomes higher than 10% as span-to-sag ratio is greater than 45 and less than 80. The largest relative error is around 12% when span-to-sag ratio increases to 60.
As stated in section 3.1, the cables are working only in tension. According to linear results as shown in Figure 5.32, when the span-to-sag ratio is larger than 25, the horizontal components of cable forces in bottom chord obtained by linear analysis are less than 0, while the nonlinear results are still greater than 0. And it is observed that the linear analysis not works in this situation.

Besides, it is obvious that the horizontal components of cable forces in bottom chord obtained by nonlinear analysis are always larger than the results obtained by linear analysis as the span-to-sag ratio changes. As shown in Figure 5.33, the largest error for Hb of linear cable truss becomes higher than 10% when the span-to-sag ratio is larger than 15. The largest one is around 660% as span-to-sag ratio increases up to 57.5 which is quite significant.
In Figure 5.34, the mid-span vertical deflections obtained by linear analysis are larger than nonlinear results at first, but when the span-to-sag ratio becomes greater than 65, the linear results start to be less than nonlinear results. It can be inferred that the stiffness in the nonlinear analysis is higher than that in linear analysis at first, and then the stiffness in the nonlinear analysis will tend to be smaller than that in linear analysis when the top cable chord becomes flatter. Further, as shown in Figure 5.35, the relative error of $w$ of linear cable truss becomes higher than 10% as span-to-sag ratio is in the range from 20 to 35. The largest relative error is around 12% as the span-to-sag ratio equals 20.
5.2.2 Asymmetric Cable Truss under Uniformly Distributed Vertical Load
\( q = 10 \text{kN/m from } x = 20 \text{m to } x = 40 \text{m} \)

The geometry of the biconcave asymmetric cable truss under a uniformly distributed vertical load applied along the span from \( x = 20 \text{m to } x = 40 \text{m} \) is shown in Figure 5.36.

Figure 5.36 Geometry of a Biconcave Asymmetric Cable Truss under Uniformly Distributed Vertical Load along Part of the Span

Figure 5.37 Horizontal Components of Cable Forces in Top Chord of Asymmetric Cable Truss Obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios
In Figure 5.37, the horizontal components of cable forces in top chord obtained by both linear and nonlinear analyses tend to be larger as the span-to-sag ratio increasing. As shown in Figure 5.38, the error of Ht of linear cable truss becomes smaller at last as span-to-sag ratio increases. The largest relative error is about 8% when span-to-sag ratio equals 2.5.

Figure 5.38 Relative Errors for Ht of Linear Cable Truss vs. Span-to-sag Ratios

Figure 5.39 Horizontal Components of Cable Forces in Bottom Chord of Asymmetric Cable Truss Obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios
From Figure 5.39, the horizontal components of cable forces in bottom chord obtained by linear analysis are always smaller than nonlinear results as span-to-sag ratio changes. As shown in Figure 5.40, all the relative errors are higher than 10%. The largest one is around 24% when span-to-sag ratio increases up to 37.5.

Figure 5.41 Mid-span Vertical Deflections of Asymmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios
In Figure 5.41, the mid-span vertical deflections obtained by linear analysis are larger than nonlinear results at first, but when the span-to-sag ratio becomes greater than 55, the linear results start to be less than nonlinear results. It can be inferred that the stiffness in the nonlinear analysis is higher than that in linear analysis at first, and then the stiffness in the nonlinear analysis tends to be less than that in linear analysis when the top cable chord becomes flatter. Further, as shown in Figure 5.42, the relative errors of linear results are higher than 10% when the span-to-sag ratio is less than 30. The largest relative error is around 40% as span-to-sag ratio equals 5.
5.2.3 Asymmetric Cable Truss under Uniformly Distributed Vertical Load $q=10$ kN/m over Left Half Span

The geometry of the biconcave asymmetric cable truss under a uniformly distributed vertical load applied over left half span is shown in Figure 5.43.

![Figure 5.43 Geometry of a Biconcave Asymmetric Cable Truss under Uniformly Distributed Vertical Load over Left Half Span](image)

Figure 5.43 Geometry of a Biconcave Asymmetric Cable Truss under Uniformly Distributed Vertical Load over Left Half Span

![Figure 5.44 Horizontal Components of Cable Forces in Top Chord of Asymmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios](image)

Figure 5.44 Horizontal Components of Cable Forces in Top Chord of Asymmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios

![Figure 5.45 Relative Errors for Ht of Linear Cable Truss vs. Span-to-sag Ratios](image)

Figure 5.45 Relative Errors for Ht of Linear Cable Truss vs. Span-to-sag Ratios
According to Figure 5.44, the horizontal components of cable forces in top chord obtained by both linear and nonlinear analyses tend to be larger as the span-to-sag ratio increases. The linear results are always less than nonlinear results. Further, as shown in Figure 5.45, the relative error for $H_t$ of linear cable truss keeps higher than 10% when the span-to-sag ratio is less than 10. The largest relative error is around 13% when span-to-sag ratio equals 2.5.

![Figure 5.46 Horizontal Components of Cable Forces in Bottom Chord of Asymmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios](image)

![Figure 5.47 Relative Errors for $H_b$ of Linear Cable Truss vs. Span-to-sag Ratios](image)

From Figure 5.46, it is obvious that the horizontal components of cable forces in bottom chord obtained by linear analysis are always smaller than nonlinear results.
Besides, as shown in Figure 5.47, all the relative errors of Hb of linear cable truss are higher than 10% as span-to-sag ratio changes. The largest one is around 33%, when span-to-sag ratio increases to 37.5.

![Figure 5.48 Mid-span Vertical Deflections of Asymmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios](image)

![Figure 5.49 Relative Errors for w of Linear Cable Truss vs. Span-to-sag Ratios](image)

In Figure 5.48, the mid-span vertical deflections obtained by linear analysis are larger than nonlinear results at first, but when the span-to-sag ratio becomes larger than 55, the linear results start to be less than nonlinear results. It can be inferred that the stiffness in the nonlinear analysis is higher than that in linear analysis at first, and then...
the stiffness in the nonlinear analysis will tend to be smaller than that in linear analysis when the top cable chord becomes flatter. Besides, as shown in Figure 5.49, the relative error for \( w \) of linear cable truss keeps higher than 10% as span-to-sag ratio is less than 30. The largest relative error is around 600% which is quite significant when span-to-sag ratio equals 2.5.

**5.2.4 Asymmetric Cable Truss under Vertical Point Load \( P=200 \text{ kN} \) at \( x=30\text{m} \)**

The geometry of the biconcave asymmetric cable truss under a vertical point load applied in the middle of the cable span is shown in Figure 5.50.

![Figure 5.50 Geometry of a Biconcave Asymmetric Cable Truss under Vertical Point Load](image)

![Figure 5.51 Horizontal Components of Cable Forces in Top Chord of Asymmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios](image)
According to Figure 5.51, the horizontal components of cable forces in top chord obtained by both linear and nonlinear analyses tend to be larger as the span-to-sag ratio increases. The linear results are always less than nonlinear results. As shown in Figure 5.52, the relative error of Ht of linear cable truss keeps higher than 10% as span-to-sag ratio is less than 45 and eventually it becomes smaller as span-to-sag ratio continuously increases. The largest relative error is around 15% as span-to-sag ratio equals 2.5.

Figure 5.53 Horizontal Components of Cable Forces in Bottom Chord of Asymmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios
From Figure 5.53, it is seen that the horizontal components of cable forces in bottom chord obtained by linear analysis are always smaller than nonlinear results. Besides, as shown in Figure 5.54, all the relative errors of $H_t$ of linear cable truss are larger than 10% as span-to-sag ratio changes. The largest one is around 36%, when span-to-sag ratio increases to 40.

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**Figure 5.54 Relative Errors for $H_b$ of Linear Cable Truss vs. Span-to-sag Ratios**

**Figure 5.55 Mid-span Vertical Deflections of Asymmetric Cable Truss obtained by Linear and Non-linear Analyses vs. Span-to-sag Ratios**
In Figure 5.55, at first, the mid-span vertical deflections obtained by linear analysis are larger than nonlinear results, but when the span-to-sag ratio becomes larger than 65, the linear results start to be less than nonlinear results. It can be inferred that the stiffness in the nonlinear analysis is higher than that in linear analysis at first, and then the stiffness in the nonlinear analysis tends to be smaller than that in linear analysis when the top cable chord becomes flatter. Further, as shown in Figure 5.56, the relative error of linear cable truss keeps higher than 10% when the span-to-sag ratio is less than 40. The largest relative error is around 64% as span-to-sag ratio equals to 5.

From the figures in section 5.2, it can be summarized as follows.

1. For cable trusses under the entire uniformly distributed vertical load, when the span-to-sag ratio is larger than 25, horizontal components of cable forces in bottom chord obtained by linear analysis are less than 0, while the nonlinear results are still greater than 0. Hence it can be seen that the linear analysis has some limitations and it can’t be used in those cases.

2. For asymmetric cable trusses, the mid-span vertical deflections obtained by both linear and nonlinear analyses tend to be larger as the span-to-sag ratio increases, and then they decrease eventually. Besides, at the beginning, the linear result is
larger than nonlinear result, but when the span-to-sag ratio increases up to around 60, the linear results start to be less than nonlinear result. It can be inferred that the stiffness in the nonlinear analysis is higher than that in linear analysis at first, and then the stiffness in the nonlinear analysis will become less than that in linear analysis.

3. Through comparison between the horizontal components in both bottom and top chords, deviation between resulting horizontal components of the cable forces in the bottom chord obtained by the linear and nonlinear analyses are more obvious than those in the top chord. Therefore, the nonlinearity of the bottom chord is more significant than that in the top chord. In this case, nonlinear cable truss theory is more necessary for calculating the horizontal tension force components in bottom chord of the cable truss for relatively more accurate result.

4. For asymmetric cable trusses under various loads, the differences between horizontal tension components of cable forces in both bottom and top chords obtained by linear and nonlinear analyses finally tend to decrease as span-to-sag ratio increases. For the horizontal tension force components in bottom chord, the largest relative errors of linear results of linear cable truss for each case are all around 10% as span-to-sag ratio changes. For the horizontal tension force components in top chord, the largest relative errors of linear results of linear cable truss for each case are greater than 25% as span-to-sag ratio changes. Regard to the mid-span vertical deflections obtained by both linear and nonlinear analyses, the largest relative errors for each case are higher than 10% and some even increase up to 600%. Therefore, the nonlinear cable truss theory is must be performed in order to get relatively accurate results.
5.3 Different Loads Applied on Symmetric Cable Trusses

In this section, different loads are given for cable trusses under four different load types in numerical analysis.

The geometric and material properties of the cable trusses are given as follows. The length of the cable truss span is \( l = 60m \). The Young’s moduli of elasticity of both cables are \( E_b = E_t = 1.5 \times 10^{11} N/m^2 \). The cross-sectional areas of the bottom and top cables are \( A_b = 1.3 \times 10^{-3} m^2 \) and \( A_t = 2.0 \times 10^{-3} m^2 \). For symmetric trusses, the initial horizontal components of pretension in the bottom and top chord are \( H_{0b} = H_{0t} = 600kN \). Further, the following data are also specified: \( d_b = d_t = 0.5m \) and \( b_b = b_t = 4.52m \). According to the relationship \( l = l/c \times (b_b - d_b) = l/s \times (b_t - d_t) \), \( l/c = l/s = 15 \).

Twelve different loading values are considered. Horizontal components of cable forces in bottom and top chords as well as the mid-span vertical deflections under applied load versus the span-to-sag ratios of the carrying cable truss obtained by the presented coupled cubic non-linear cable truss equations are compared with those obtained by the coupled linear cable truss equations.

5.3.1 Cable Truss under Various Uniformly Distributed Vertical Loads

The geometry of the biconcave cable truss under a uniformly distributed vertical load applied over entre span is shown in Figure 5.57.
Figure 5.57 Geometry of a Biconcave Cable Truss under Various Uniformly Distributed Vertical Load

Figure 5.58 Horizontal Components of Cable Forces in Top Chord of Cable Truss obtained by Linear and Non-linear Analyses vs. Loads

Figure 5.59 Relative Errors for Ht of Linear Cable Truss vs. Loads
In Figure 5.58, the horizontal components of cable forces in top chord obtained by nonlinear analysis are always greater than linear results, and the differences between them tend to be larger as load increasing. As shown in Figure 5.59, the largest relative error for Ht of linear cable truss is about 1.2% when load equals 15kN/m.

![Figure 5.60](image)

**Figure 5.60** Horizontal Components of Cable Forces in Bottom Chord of Cable Truss obtained by Linear and Non-linear Analyses vs. Loads

In Figure 5.60, the horizontal components of cable forces in bottom chord obtained by nonlinear analysis are always greater than linear results, and the differences between them tend to be larger as load increasing. As shown in Figure 5.61, the relative error for Hb of linear cable truss keeps higher than 10% when load is larger than 10kN/m. The largest one is about 100% when load equals 15kN/m.

![Figure 5.61](image)

**Figure 5.61** Relative Errors for Hb of Linear Cable Truss vs. Loads
In Figure 5.62, the mid-span vertical deflections obtained by nonlinear analysis are always less than linear results, and the differences between them tend to be larger as load increasing. As shown in Figure 5.63, the largest relative error for $w$ of linear cable truss is about 4.5% when load equals 15kN/m.
5.3.2 Cable Truss under Various Uniformly Distributed Vertical Loads from x=20m to x=40m

The geometry of the biconcave cable truss under a uniformly distributed vertical load applied along the span from x=20m to x=40m is shown in Figure 5.64.

Figure 5.64 Geometry of a Biconcave Cable Truss under Uniformly Distributed Vertical Load along part of the Span

Figure 5.65 Horizontal Components of Cable Forces in Top Chord of Cable Truss obtained by Linear and Non-linear Analyses vs. Loads
In Figure 5.65, the horizontal components of cable forces in top chord obtained by nonlinear analysis are always greater than linear results, and the differences between them tend to be larger as load increasing. As shown in Figure 5.66, the relative error for Ht of linear cable truss keeps higher than 10% when load is larger than 20kN/m. The largest one is about 12% when load equals 30kN/m.

Figure 5.67 Horizontal Components of Cable Forces in Bottom Chord of Cable Truss obtained by Linear and Non-linear Analyses vs. Loads
In Figure 5.67 the horizontal components of cable forces in bottom chord obtained by nonlinear analysis are always greater than linear results, and the differences between them tend to be larger as load increasing. As shown in Figure 5.68, the relative error for $H_b$ of linear cable truss keeps higher than 10% when load is larger than 10kN/m. The largest one is about 100% when load equals 30kN/m.

Figure 5.69 Mid-span Vertical Deflections of Cable Truss obtained by Linear and Non-linear Analyses vs. Loads
In Figure 5.69, the mid-span vertical deflections obtained by nonlinear analysis are always less than linear results, and the differences between them tend to be larger as load increasing. As shown in Figure 5.70, the largest relative error for w of linear cable truss is about 40% when load equals 30kN/m, and the errors keep higher than 10% when loads are greater than 10kN/m.

5.3.3 Cable Truss under Various Uniformly Distributed Vertical Loads over Left Half Span

The geometry of the biconcave cable truss under a uniformly distributed vertical load applied over left half span is shown in Figure 5.71.
In Figure 5.72, the horizontal components of cable forces in top chord obtained by nonlinear analysis are always greater than linear results, and the differences between them tend to be larger as load increasing. As shown in Figure 5.73, the relative error for Ht of linear cable truss keeps higher than 10% when load is larger than 12.5kN/m. The largest one is about 19% when load equals 30kN/m.
Figure 5.74 Horizontal Components of Cable Forces in Bottom Chord of Cable Truss obtained by Linear and Non-linear Analyses vs. Loads

In Figure 5.74 the horizontal components of cable forces in bottom chord obtained by nonlinear analysis are always greater than linear results, and the differences between them tend to be larger as load increasing. As shown in Figure 5.75, the relative error for Hb of linear cable truss keeps higher than 10% when load is larger than 7.5kN/m. The largest one is about 100% when load equals 30kN/m.
In Figure 5.76, the mid-span vertical deflections obtained by nonlinear analysis are always less than linear results, and the differences between them tend to be larger as load increasing. As shown in Figure 5.77, the largest relative error for \(w\) of linear cable truss is about 22% when load equals 30kN/m, and the errors keep higher than 10% when loads are greater than 10kN/m.
5.3.4 Cable Truss under Various Vertical Point Loads at x=30m

The geometry of the biconcave cable truss under a vertical point load applied in the middle of the cable span is shown in Figure 5.78.

![Figure 5.78 Geometry of a Biconcave Cable Truss under Vertical Point Load](image)

Figure 5.78 Geometry of a Biconcave Cable Truss under Vertical Point Load

![Figure 5.79 Horizontal Components of Cable Forces in Top Chord of Cable Truss obtained by Linear and Non-linear Analyses vs. Loads](image)

Figure 5.79 Horizontal Components of Cable Forces in Top Chord of Cable Truss obtained by Linear and Non-linear Analyses vs. Loads

![Figure 5.80 Relative Errors for Ht of Linear Cable Truss vs. Loads](image)

Figure 5.80 Relative Errors for Ht of Linear Cable Truss vs. Loads
In Figure 5.79, the horizontal components of cable forces in top chord obtained by nonlinear analysis are always greater than linear results, and the differences between them tend to be larger as load increasing. As shown in Figure 5.80, the relative error for Ht of linear cable truss keeps higher than 10% when load is larger than 200kN. The largest one is about 21% when load equals 600kN.

![Figure 5.81 Horizontal Components of Cable Forces in Bottom Chord of Cable Truss obtained by Linear and Non-linear Analyses vs. Loads](image1)

![Figure 5.82 Relative Errors for Hb of Linear Cable Truss vs. Loads](image2)

In Figure 5.81 the horizontal components of cable forces in bottom chord obtained by nonlinear analysis are always greater than linear results, and the differences between them tend to be larger as load increasing. As shown in Figure 5.82, the relative error for Hb of linear cable truss keeps higher than 10% when load is larger than 150kN. The largest one is about 100% when load equals 600kN.
In Figure 5.83, the mid-span vertical deflections obtained by nonlinear analysis are always less than linear results, and the differences between them tend to be larger as load increasing. As shown in Figure 5.84, the largest relative error for w of linear cable truss is about 78% when load equals 600kN, and the errors keep higher than 10% when loads are greater than 100kN.

From the figures in section 5.3, it can be summarized as follows.

1. For horizontal components of the cable force both in top and bottom chords and mid-span vertical deflections, the difference between linear and nonlinear results
tends to be larger as load increasing. It can be inferred that the nonlinear behavior of the cable truss structures is more significant when load is increasing.

2. The mid-span vertical deflections obtained by linear analysis are always larger than nonlinear results and it means that the stiffness in the nonlinear analysis is higher than it in linear analysis.

3. Through comparison between the horizontal components in both bottom and top chords, deviation between resulting horizontal components of the cable forces in the bottom chord obtained by the linear and nonlinear analyses are more obvious than those in the top chord. Therefore, the nonlinearity of the bottom chord is more significant than that in the top chord. In this case, nonlinear cable truss theory is more necessary for calculating the horizontal tension force components in bottom chord of the cable truss for more accurate result.
Chapter 6 Conclusions and Recommendations

6.1 Conclusions

Based on the numerical analysis, investigations, and discussions in chapter 5, the following conclusions can be given.

1. For symmetric cable truss under the entire uniformly distributed vertical load, when the span-to-sag ratio is larger than 25 and less than 80, the horizontal components of bottom cable forces obtained by linear analysis become less than 0. However, the nonlinear results are still greater than 0. For asymmetric cable truss under the entire uniformly distributed vertical load, when the span-to-sag ratio is larger than 25, the linear horizontal tension force components in bottom chord are less than 0, whereas the nonlinear results are still greater than 0. Hence the linear analysis has some limitations and it can’t be used in some cases. Therefore, in those cases, the nonlinear cable truss theory must be used.

2. For symmetric cable trusses under various loads, the mid-span vertical deflections obtained by linear analysis are always greater than nonlinear results. Thus, the stiffness of the cable truss in nonlinear analysis is higher than that in linear analysis.

3. For both symmetric and asymmetric cable trusses, the deviation between resulting horizontal tension force components in the bottom chord obtained by the linear and nonlinear analyses are more obvious than those in the top chord. The nonlinearity of the bottom cable is more obvious than that of the top cable. For calculating the horizontal tension force components in bottom chord of the cable truss, using nonlinear cable truss theory is more necessary in order to get more accurate results.
4. For symmetric cable trusses, it can be found that the horizontal components of the tension forces in both top and bottom chords obtained by linear analysis are always smaller than those obtained by nonlinear analysis as span-to-sag ratio changes. The mid-span vertical deflections obtained by linear analysis are greater than nonlinear results. Therefore, although the results are a little overestimated, it is considerably conservative if linear cable truss theory is used to calculate the cable truss deflections.

5. For symmetric cable trusses, the relative errors of the horizontal components of bottom cable tension forces obtained from linear analysis are larger than 10% when span-to-sag ratios are greater than 80, and the largest relative errors for each case are all higher than 12%. Regard to the horizontal tension force components in top chord, the relative errors for each case are all larger than 10% when span-to-sag ratios are greater than 15. The highest relative errors of linear results for each case are all more than 40%. In addition, for the vertical deflections obtained by both linear and nonlinear analyses, the relative errors for each case are all larger than 10% when span-to-sag ratios are greater than 25. The largest relative errors for each case are all higher than 40% and some even increase up to 80%. Thus, the nonlinear cable truss theory must be used in these cases in order to get acceptable accurate results.

6. For asymmetric cable trusses under various loads expect that under uniformly distributed vertical load applied on the entire span, the relative errors of the linear horizontal components of top cable tension forces of linear cable truss are large especially when the span-to-sag ratios are less than 50. The largest relative errors are all around 10%. For the horizontal tension force components in bottom chord,
the relative errors are greater than 10% when the span-to-sag ratios are less than 65. The largest errors of linear results of linear cable truss for each case are all greater than 20% as span-to-sag ratio changes. In addition, regard to the mid-span vertical deflections obtained by both linear and nonlinear analyses, the relative errors are larger than 10% when the span-to-sag ratios are less than 20. The largest relative error for each case is higher than 35%, and it can increase up to 600% which is quite large for the cable truss under uniformly distributed vertical load applied over the left half span. In these cases, the nonlinear cable truss theory must be used.

7. For the asymmetric cable trusses under uniformly distributed vertical load applied on the entire span, the relative errors of the linear horizontal components of top cable tension forces of linear cable truss become large especially when the span-to-sag ratios are greater than 40 and less than 60. The largest relative error is around 12%. For the horizontal tension force components in bottom chord, the relative errors are higher than 16% when the span-to-sag ratios are larger than 15. Regard to the mid-span vertical deflections obtained by both linear and nonlinear analyses, the relative errors become great especially when the span-to-sag ratios are larger than 20 and less than 30 and the relative errors are higher than 10%. In these cases, the nonlinearity of the cable truss must be taken into account in numerical analysis.

8. For horizontal components of the cable forces both in top and bottom chords and mid-span vertical deflections, the difference between linear and nonlinear results tends to be larger as load increasing. It can be inferred that the nonlinear behavior of the cable truss structures is more significant when load is increasing.
6.2 Recommendations

1. In this thesis, only the static loadings are taken into consideration. The dynamic analysis can be considered in the future.

2. Temperature change is not investigated during numerical analysis. In the future, this factor can be considered in order to study the behavior of the cable trusses under temperature change.

3. Here the effects of the elastic cable truss deformations are only corrected to the second order. Higher order terms can be considered and investigated to show the differences of the results and different behaviors of the cable trusses.

4. Finite element method can also be used for analysis and comparisons.
References


